Brian Alspach and His Work

Joy Morris

Department of Mathematics and Computer Science
University of Lethbridge
Lethbridge, AB. T1K 6R4. Canada

Mateja Šajna

Department of Mathematics and Statistics
University of Ottawa
585 King Edward Avenue
Ottawa, ON. K1N 6N5. Canada

It's hard to think of Brian Alspach these days, without conjuring up a mental picture of feet in Birkenstocks, and his Australian hat within arm's reach, if not perched on his head. Often, a briefcase will be stashed nearby, if he isn't actually carrying it.

These outward symbols are accurate reflections of Brian's character. Although his outlook is casual and laid-back, his attention rarely strays far from his passions in life, which include the work that is stored in that briefcase.

Brian's 65th birthday, on May 29, 2003, was celebrated at the "Graph Theory of Brian Alspach" conference, held at Simon Fraser University (SFU) in Vancouver (where he spent most of his career). This volume of papers forms the proceedings of the conference.

The conference banquet provided a rare opportunity for Brian's colleagues, collaborators, and students, as well as friends and family members to get together, and to share stories about Brian. The guests were delightfully entertained by the "Brian Alspach Quiz" prepared for the occasion by Brooks Reid, who served as the Master of Ceremonies. Brooks has graciously allowed us to include some of his quiz questions in this paper. We are happy to share

Email addresses: joy@cs.uleth.ca (Joy Morris), msajna@uottawa.ca (Mateja Šajna).

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them with you. Answers either appear with the questions, or can be found in the text.

Brian Alspach has been a tremendous influence in the mathematical community. He enjoys meeting young mathematicians, and at conferences, is often to be found at the center of a very lively group. He is a talented mentor, with a knack for encouraging young people. Even a brief meeting with him has proven influential in numerous careers, but Brian has no hesitation in going far out of his way to provide further encouragement. A substantial amount of his grant money goes toward supporting students — not only his own — with funding to attend conferences or to visit other researchers. He has also regularly provided financial assistance to researchers who want to visit and work with him. His home is often opened to visitors who are at various stages of their careers, but particularly to students and to mathematicians who are still establishing themselves.

Brian's example and leadership have set the standards for collaborative research, not only among those mathematicians whom he has mentored directly, but among all of those who work in the areas where his research has been influential. In general, these areas of research do not have the cut-throat attitude that is too often seen in the scientific community. Instead, credit for results is shared generously, and assistance is offered freely. This is certainly Brian's own approach. He has collaborated with almost 60 different co-authors.

Brian maintains a keen interest in applications of mathematics, particularly those in industry, which he feels are an important part of maintaining young adults' interest in mathematics. He keeps abreast of research in areas beyond mathematics, and has published papers about applications of mathematics. He maintains contacts in industry, has done consulting work, and has led industry-funded projects. He provided the energy and leadership that recently led to the foundation of an industrial mathematics program at Simon Fraser University, as well as having founded and served as the Coordinator for the related Master's of Management and Systems Science program for many years. Brian was also one of the founders of the co-op education program at SFU; his role included championing the program through resistance at the University Senate. (In co-operative education, universities work with employers to provide students with alternating semesters of study and job experience; the program at SFU is now one of the longest-standing and most respected in Canada.)

It would be remiss to write much about Brian without mentioning ways in which he combines the passions in his life, and uses his other interests to popularise mathematics. He enjoys giving talks to the public and to school groups, and when doing so, he often uses either sports or games to introduce mathematical problems in an interesting way. He has a great interest in both football (specifically, the Washington Huskies), and baseball. Besides being

Question 1. Brian was, at one time, an avid:

- *a) fisherman*
- *b) skier*
- *c) horse shoer*
- *d) golfer*
- *e) softball player*
- *f) curler*

Question 2. Brian is an ardent fan of:

- *a) Saskatchewan Roughriders football*
- *b) U. Washington Husky football*
- *c) San Diego Padres baseball*
- *d) New Westminster Salmonbellies Box lacrosse*
- *e) New Zealand All-Blacks rugby*
- *f) B.C. Lions football*

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a fan of the Seattle Mariners baseball team, he played on fast pitch softball teams in leagues around the Vancouver area for many years. More recently, his enjoyment of poker has led him to writing regular articles in popular poker magazines, about the mathematics behind various situations in that game. He has also done consulting work in the gaming industry.

This paper consists of a brief overview of Brian's life, followed by discussions of some of the areas in which he has made significant research contributions. At the end of this paper is a complete list of his publications.

Brian was born on May 29, 1938, in North Dakota. By the time he was nine years old, his family had settled in Seattle, Washington. He has one brother, Neal, and one sister, Diane, both of whom live in the Seattle area, where his mother also still resides. After finishing his undergraduate degree at the University of Washington in 1961, Brian moved to California, where he undertook graduate work at the University of California – Santa Barbara. He maintained a keen interest in the Washington Huskies college football team, and had season tickets to their home games for years. When living in Vancouver, he would drive down to Seattle regularly to watch their games, and would occasionally travel to attend road games.

Brian taught one year (1961-62) of junior high school between his B.A. and his Ph.D.

Question 3. Before settling into Cayley graphs, decompositions, automorphisms, etc. Brian worked in:

- *a) Ramsey theory*
- *b) enumeration*
- *c) tournaments*
- *d) reconstruction*
- *e) matroids*
- *f) divinity*

Question 4. Brian's earliest published paper appeared in what year?

- *a) 1965*
- *b) 1967*
- *c) 1968*
- *d) 1969*
- *e) 1970*
- *f) 1938*

Brian's graduate work was done under the supervision of Paul Kelly, in a department that was very strong in the area of linear algebra. Kelly is given joint credit with Ulam for posing the famous Graph Reconstruction Conjecture, and established the result for trees. Brian completed his M.A. at UCSB in 1964, and his Ph.D. in 1966; the title of his thesis was "A Class of Tournaments."

Brian moved to Vancouver in 1966, where he joined Simon Fraser University's Department of Mathematics and Statistics one year after the university was founded. During his career there, he was influential at a wide variety of levels, from the local to the international. He sat on editorial boards of journals, helped to organise international conferences and workshops, and wrote many Math Reviews.

Question 5. Brian was a host for the "Cycles in Graphs" 8-week summer workshop/conference at SFU in:

a) 1982
b) 1980
c) 1978
d) 1986
e) 1984
f) 1988

Question 6. Brian was a problem editor for

a) Discrete Mathematics
b) Journal of Comb. Theory
c) Journal of Graph Theory
d) Bulletin of the Canadian Math. Soc.
e) Spectrum
f) Backgammon Today

Answers. The highly successful "Cycles in Graphs" workshop, organised by Brian and Pavol Hell, took place in 1982. It included very generous time for work and collaboration. The proceedings were edited by Brian, with Chris Godsil, and appeared as a volume in *Annals of Discrete Math.* Brian also contributed many open problems to *Discrete Mathematics.*

At a regional level, he helped to found, and was the main coordinator for the West Coast "Combinatorial Potlatches," from their origins in the 1970s, until 1997. These one-day gatherings, held once or twice a year, bring together combinatorial researchers and students from around the Pacific Northwest, to discuss their research interests and to get to know one another.

Locally, he organised and coordinated seminars, mentored students, and participated in many projects to raise awareness of mathematics in the community. By the late 1970s, the graduate program in discrete mathematics at SFU had grown to the point where Brian decided it was worthwhile to institute an instructional seminar series. This series has been held on a weekly basis during the academic year, ever since, and has exposed participants to a wide variety of current-interest topics in discrete mathematics. Brian also played an important role in attracting other combinatorialists to the department, and was instrumental in the recruitment and hiring of several, including Pavol Hell, Chris Godsil, Kathy Heinrich, and Luis Goddyn.

Brian had been married to Linda in 1961. After they split up, Brian met Kathy Heinrich at the *Southeastern International Conference on Combina-*

Question 7. Brian missed a discrete mathematics conference at UBC, a conference he organized, because he took off for:
  a) Mississippi
  b) Italy
  c) Cayman Islands
  d) Fiji
  e) Singapore
  f) Australia

Question 8. Brian's wife is also a mathematician who works in:
  a) Algebraic Number Theory
  b) Logic
  c) P.D.E.'s
  d) Combinatorics
  e) The History of Zero
  f) Numerical Analysis

Answers. The answer to Question 7 is Australia, the native country of Brian's wife, Kathy Heinrich. She is a combinatorialist who received her Ph.D. from the University of Newcastle under the supervision of Walter Wallis.

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torics, Graph Theory and Computing, in 1977. Brian and Kathy were married in 1980, in Reno, but kept it very quiet; many members of the department speculated for years about the status of their relationship. Kathy joined the Department of Mathematics and Statistics at SFU in 1981.

Frequent visitors passed through Brian and Kathy's home on the north shore of Burrard Inlet, from SFU students and colleagues attending a summer barbecue, to research collaborators young and old who came for a longer stay. Many of us have fond memories of summer afternoons when Brian took time off to cook. A meal might include such favourites as barbecued wild salmon (sockeye, to be precise — the tastiest kind according to Brian), Brian's special fresh spinach salad with grapes and mushrooms (for which the early guests themselves could help pick tender young spinach leaves in the large vegetable garden), and Brian's signature dessert, the inevitable scrumptious blueberry pie. Brian was a committed gardener, and his produce was always a source of pride to him. On many other occasions, meals at Brian and Kathy's home were prepared cooperatively by several mathematicians (including visitors, locals, students, and former students). Brian was also known for organising salmon bakes on the beach at Spanish Banks in Vancouver.

After a dinner at their home, guests might admire Brian's extensive collection of jazz and classical music recordings, and be introduced to some of his favourites. Some would vie for the chance to play a game of pinball on the legendary machine standing in the basement, dreaming of scoring 100,000 points, and thus winning the coveted honour of having their names inscribed in Brian's book. Others might play with Brian and Kathy's friendly cat if he happened to be around, and — if they noticed it at all — muse over the small ancient TV set hidden in a dark faraway corner of the house. These traditions continued after Brian and Kathy tore down their old home, and built a beautiful new one on the same site, filled with art work they had brought from their travels around the world.

Question 9. One of Brian's students went on to be a:

- *a) famous Toronto chef*
- *b) high financier*
- *c) Canadian table tennis senior champion*
- *d) slum lord*
- *e) fireman*
- *f) Provincial Attorney General*

Question 10. Which pair of initials is ***not*** *a pair of initials of Brian's students:*

- *a) C.Q.Z. and S.M.*
- *b) J.M. and J.L.*
- *c) L.G. and K.H.*
- *d) B.V. and M.S.*
- *e) S.M. and L.V.*
- *f) J.A. and S.Z.*

Answers. Brian indeed counts a high financier among his former students; however, Luis Goddyn and Kathy Heinrich, who are hiding behind the initials L. G. and K. H. in Question 10, are not among them.

Brian spent 32 years at Simon Fraser University before taking early retirement in September of 1998, at the age of 60. His retirement left him free to move to Regina, Saskatchewan in August of 1999, when Kathy was offered the position of Vice-President Academic at the University of Regina. Brian now holds an adjunct position at the University of Regina. After the move, his remaining graduate students divided their time between Vancouver and Regina. All of his Master's students have since graduated; the last of his 13 Ph.D. students has successfully defended his thesis, and Brian maintains that he will not be taking on any more students. He has gradually been divesting himself of teaching, administrative, and administrative research-related commitments, over the last few years.

Since moving to Regina, Brian has instituted annual "Prairie Discrete Math Workshops," similar in nature to the Combinatorial Potlatches, but covering the Prairies region and lasting for a weekend.

Question 11. Brian publishes in:

- *a) The Journal of Improbable Results*
- *b) Better Homes & Gardens*
- *c) College Math. Journal*
- *d) Poker Magazine*
- *e) Mechanics Illustrated*
- *f) Playboy*

Question 12. During the Southeastern International Conference in Florida, Brian often frequented:

- *a) high end shopping malls*
- *b) Little Havana in Miami*
- *c) greyhound racing*
- *d) Greyhound bus station*
- *e) New York Yankee spring baseball*
- *f) Jai Alai*

Answers. Jai Alai is another sport that has interested Brian, and he often took the opportunity of seeing it played professionally in Florida, with bets on outcomes adding to the excitement.

To some extent, Brian has filled the gap left by his retirement with other pursuits. Shortly before retirement, he began to take piano lessons; he particularly enjoys both classical and jazz music, and has a long-standing appreciation for both opera (he would drive from Vancouver to Seattle for certain operatic productions, as well as for Husky games) and the symphony. His interest in poker has intensified, and he writes regular articles about the mathematics of poker for both the *Poker Digest* and the *Canadian Poker Player* magazine; copies of these articles are available from his web page, `http://www.math.sfu.ca/~alspach`.

He tells us that soon, he will have finished shedding other responsibilities, and will be free to concentrate on some of the research problems that continue to particularly intrigue him. Another motivation for clearing his plate is to leave himself free to travel as he pleases.

In addition to his mathematical achievements, Brian is the proud father of two children (by his marriage to Linda), Alina and Mark; he has four granddaughters.

Brian came onto the graph theory scene just as the first books in English on the subject appeared. Thus, his career has spanned the tremendous development of the subject, and his work and influence have contributed significantly to that development. His research has been ground-breaking in a number of areas within graph theory that have since attracted a great deal of attention and interest. Summaries of some of Brian's major contributions to research follow. A full list of his publications can be found at the end of this paper.

Notation. In all that follows, p and q will be used exclusively to denote distinct prime numbers; if a number could be composite, another letter, such as k, m, or n, will be used. The letter G will always denote a group, and X and Y will always denote graphs.

1 Tournaments and Digraphs

Brian's thesis, and many of his early papers, dealt with tournaments: simple digraphs without digons, whose underlying graph is complete. Some of these results generalise to broader classes of directed graphs. Although Brian gradually moved away from tournaments in his research, his work on tournaments was fundamental to a number of research topics in this area. In Brian's first published paper [1], he proved that regular tournaments are arc-pancyclic: that is, that for every arc in a regular tournament and every length k between 3 and n, where n is the number of vertices of the tournament, the specified arc appears in a cycle of length k. This work initiated the major research direction of Hamilton connectivity in directed graphs, which is now a fruitful subject area with hundreds of papers. It also formed the basis and motivation for many related results about the cycle structure of tournaments.

Several of Brian's early results deal with symmetries of tournaments, reflecting his developing interest in permutation groups. With Myron Goldberg and John Moon, he proved an analogue for tournaments of a result of Sabidussi, determining the automorphism group of the wreath product of two tournaments in terms of the automorphism groups of the two tournaments [2]. He found a combinatorial proof of a previously-known upper bound on the order of the automorphism group of any tournament on n vertices [3], and later improved this by coming up with a recursive formula for the exact value of the maximum order of such an automorphism group [6], in joint work with Len Berggren.

A tournament (necessarily of odd order) is called a circulant tournament if its automorphism group contains a transitive cyclic subgroup. Brian proved [4] that a circulant tournament is always self-converse: that is, there is a permutation on the vertices that reverses every arc in the tournament. He then showed that any self-converse vertex-transitive tournament is a circulant tournament, but that vertex-transitivity by itself is not sufficient. Although this result has not attracted a great deal of attention, it has a strong flavour of results that he later obtained on circulant graphs, and these have led to much additional research. Some of Brian's early results [10,14] involved path decompositions of digraphs and tournaments. These were joint work with Norman Pullman and David Mason. They represent Brian's first works in the area of decompositions, where some of his most significant research has been done.

If there is an arc from u to v in an asymmetric digraph, then a directed path of length k from u to v is called a k-bypass. An asymmetric digraph D that is not totally disconnected is said to have the 2-bypass property, if for every arc (u, v) of D, there is a 2-bypass from u to v. Brian, Brooks Reid and David Roselle showed [9] that for every $n \geq 7$, there is a strongly connected asymmetric digraph of order n with the 2-bypass property. Furthermore, if the k-bypass property is defined analogously, then any asymmetric digraph with the 2-bypass property also has the 3-, 4-, 5-, and 6-bypass properties. Finally, of all labelled asymmetric digraphs on n vertices, the proportion that have the 2-bypass property tends to 1 as n tends to infinity. These results led to a great deal of work on connectivity properties of tournaments.

The number of vertices in a tournament having a particular score is called the frequency of that score. The set of frequencies of scores of a tournament is called the (score) frequency set of the tournament. Brian and Brooks Reid [18] proved that each nonempty set of positive integers is the frequency set of some tournament, and they determined the smallest possible order for such a tournament. They also gave a similar result for digraphs.

2 Permutation Groups and Their Actions on Graphs

Brian has been working on problems regarding the interactions of graphs with permutation groups for many years. He is widely regarded as an expert in this area, and particularly on the topic of Cayley graphs. The class of Cayley (di)graphs is a restriction of the class of vertex-transitive (di)graphs, to the case where the automorphism group of such a graph contains a regular subgroup. (A permutation group action on a set Ω is regular if for any $u, v \in \Omega$, there is a unique element g of the group, for which $g(u) = v$.) If G is a regular subgroup of $\mathrm{Aut}(X)$, we say that X is a Cayley (di)graph on G. This restriction from vertex-transitive graphs to Cayley graphs makes many problems easier to solve, and eliminates the need to consider some pesky recurring counter-examples, such as the Petersen graph. Brian has written chapters about Cayley graphs for several books, including the *Handbook of Graph Theory* [76], and *Topics in Algebraic Graph Theory* [77]. Some of his work in this area is outlined below.

2.1 Automorphism groups of graphs

In a 1967 paper, Turner [*J. Combinatorial Theory* **3** (1967), 136–145] showed that any vertex-transitive graph on p vertices is a circulant graph (that is, a Cayley graph on a cyclic group). This paper was a major influence on Brian; it was the first source of his lasting interest in automorphisms of graphs. In 1973, Brian determined the automorphism group for such a graph or digraph. In fact, he gave an explicit method for determining the automorphism group, from the standard representation for such a circulant (di)graph. He also used this to enumerate the number of circulant (di)graphs of order p that have a fixed automorphism group [5,7]. This work leads easily to a polynomial-time algorithm for determining the full automorphism group of a circulant (di)graph on p vertices. Subsequently, other authors built on this work to produce considerably more complicated structural results about the automorphism groups of circulant (di)graphs on n vertices, in the cases where n is either a prime power, or square-free.

One way of learning about the automorphism group of a graph, is to determine its transitivity properties. This was an approach that interested Brian, and he had several results in this area. In 1973, Brian gave a new proof [8] of a result characterising finite permutation groups G whose action is 2-homogeneous (i.e., transitive on unordered pairs but not 2-transitive). This was closely related to his later work on 1/2-transitive graphs: that is, graphs whose automorphism group acts transitively on the vertices and on the edges, but not on the arcs. These graphs were first considered by Bouwer in 1970, but were almost completely neglected until Brian's paper [59] with Dragan Marušič and Lewis Nowitz, in which the first infinite family of 1/2-transitive (metacirculant) graphs of a fixed degree (namely, 4) was constructed. Soon followed another paper [61], this time with Ming-Yao Xu, in which Brian determined all 1/2-transitive graphs on $3p$ vertices, where $p \neq 2$. The subject quickly gained in popularity and has since been generating a number of papers every year. Brian's results have been used by other researchers studying 1/2-transitive, as well as 1-regular graphs (that is, graphs whose automorphism group acts regularly on the arcs).

Brian, working with Marston Conder, Dragan Marušič, and Ming-Yao Xu, was also able to determine precisely which circulant graphs are k-arc-transitive for $k \geq 2$: that is, have automorphism groups that act transitively on the directed walks of length at least 2 that do not double back on themselves [65]. Their characterisation shows that very few classes of circulant graphs are highly symmetric in this sense. There have been a number of subsequent papers that extend their result towards a characterisation of circulant graphs whose automorphism groups act transitively on the arcs.

In 1974, Brian posed the following problem about automorphism groups [11]: for each subgroup of the symmetric group S_n, determine whether or not it is the automorphism group of some graph whose vertex set is $\{1, \ldots, n\}$. At the time, he was able to solve the problem for $n \leq 7$. Like the Graphical Regular Representation (GRR) question, this question hit on an important problem with Frucht's 1938 construction, for any abstract group G, of a graph whose isomorphism group is abstractly isomorphic to G. Although there are many partial solutions, in its full generality this problem of Brian's remains open.

2.2 The CI-problem

Another area in which Brian produced seminal research is known as the CI-problem, which is the problem of determining which graphs and which groups have the CI-property (defined shortly). The main significance of this property is two-fold: it greatly simplifies the problem of testing for isomorphisms between graphs, and it also makes enumeration of non-isomorphic Cayley graphs on a group with the CI-property much more feasible. Brian made the first real progress on the latter problem, when he and Marni Mishna determined the number of non-isomorphic Cayley graphs (and digraphs) on every cyclic group that has the CI-property, as well as certain other classes of Cayley (di)graphs that have the CI-property [73]. Both of these problems (isomorphism testing and enumeration of isomorphism classes) are extremely difficult in general, and finding solutions for Cayley graphs in particular is important, because of their extensive use in network theory. A Cayley graph X on the group G has the CI-property if, whenever Y is isomorphic to X, there is a group automorphism of G that induces a graph isomorphism from X to Y in a natural way. A group G has the CI-property if every Cayley graph on G has the CI-property. Although much work has been done on the CI-problem and many significant partial results have been obtained (the groups for which the problem remains open are in a quite limited list), the problem remains unsolved.

Brian's results on this problem include a proof that the cyclic group of order pq has the CI-property, and a determination of precisely which circulant graphs of order p^2 have the CI-property [19]; these results were obtained in joint work with Tory Parsons. After many subsequent partial results, cyclic groups were eventually classified according to which have the CI-property, but [19] was the first significant step in this effort. In the same paper, they also observed that if there is a connected Cayley graph on the group G that does not have the CI-property, then for any group G' for which $G \leq G'$, there is a connected Cayley graph on G' that does not have the CI-property.

For elementary abelian groups, Brian and Lewis Nowitz found elementary proofs that $\mathbb{Z}_p^2$ and $\mathbb{Z}_p^3$ have the CI-property [70]. Although these results were previously known, there is some hope that their technique can be extended, at least to the first unknown case, $\mathbb{Z}_p^5$. It is currently known that $\mathbb{Z}_p^n$ does not have the CI-property for n sufficiently large, but the only prime for which all elementary abelian groups have been characterised according to which have the CI-property, is $p = 2$.

2.3 Other work on permutation groups acting on graphs

In a very important 1982 paper, Brian and Tory Parsons introduced "metacirculant" graphs [31], an infinite family of vertex-transitive graphs that includes the Petersen graph. All of these graphs contain a vertex-transitive metacyclic group in their automorphism groups. Partly because they include the Petersen graph, these graphs are a very important and natural family of vertex-transitive graphs, and numerous researchers have studied them in relation to a variety of problems about vertex-transitive graphs. In some sense, they are the simplest infinite family of vertex-transitive graphs that involve non-abelian group actions. They are a rich source of many interesting families of graphs, such as vertex-transitive graphs that are not Cayley graphs, and 1/2-transitive graphs. Metacirculant graphs are a generalisation of the concept of "2-circulant" graphs, introduced in an earlier paper of Brian's [20] that was written with Richard Sutcliffe.

Brian has also produced noteworthy results on 3-edge-colourability of Cayley graphs together with Yi-Ping Liu and C.Q. Zhang [66], and has written a paper applying Cayley graphs to a problem in computer science [57].

3 Decompositions and Factorisations

Brian's interest in graph decompositions and factorisations has spanned more than three decades, and seems to have grown only stronger over the years. It started, as mentioned earlier, with two papers [10,14] on path decompositions of digraphs, and has since encompassed topics such as cycle and hamiltonian decompositions, orthogonal factorisations, 1-factorisations, and more. Out of all of his work in this area, two items have been particularly outstanding. First, he made one of the most significant contributions to the complete solution of the Oberwolfach problem for equal cycle lengths, a problem that had attracted much interest for almost 20 years before Brian began to work on it. Second, more recently, he developed powerful techniques that have led to the complete solution of the old problem of decomposing complete and nearly complete graphs into cycles of a fixed length. This problem goes back more than a century, and has been spun off into many related areas of research, but Brian's work on it represented an enormous break-through that cracked the problem wide open.

A graph will be called nearly complete, and denoted by $K_n - I$, if it is obtained from the complete graph K_n of even order n by removing the edges of a 1-factor. A decomposition of a graph X into its subgraphs $X_1, \ldots, X_k$ is a partition of the edge set of X into the edge sets of $X_1, \ldots, X_k$. A Y-decomposition of a graph X is a decomposition of X into subgraphs isomorphic to Y.

3.1 Cycle decompositions of complete and nearly complete graphs

It is easy to determine the following necessary conditions for K_n to admit a decomposition into cycles of length m: the degree $n-1$ of the graph must be even and the cycle length m must divide the number of edges $n(n-1)/2$. The conjecture that these necessary conditions are also sufficient shares its origins with the first results on the existence of Steiner triple systems in the mid-nineteenth century. The first positive results of a general type (in particular, for m even and $n \equiv 1 \pmod{m}$) were obtained in the 1960s, and over the following decades, a concerted effort was invested into this problem by many mathematicians. Brian's first contribution was a 1980 paper [25] with Badri Varma, where the conjecture is shown to be true for cycle length twice a prime power. A good decade later, he extended the conjecture to include nearly complete graphs, thus claiming that K_n with n odd, and $K_n - I$ with n even, can be decomposed into cycles of length m if and only if m divides

$n(n-1)/2$ and $n(n-2)/2$, respectively. In a paper with Susan Marshall [62] he showed that the conjecture is true if n is divisible by 4 and congruent modulo m to some k with $m/2 \leq k < m$. By 1989, Hoffman, Lindner, and Rodger [*J. Graph Theory* **13** (1989), 417–426] had developed the so-called reduction step, showing that in the case with m and n both odd, it is sufficient to prove the conjecture for n in the interval $[m, 3m)$. Relying on this result and working with Heather Gavlas, Brian discovered a new approach for constructing m-cycle decompositions of complete graphs of order n in the case where m and n are both odd, and nearly complete graphs of order n in the case where m and n are both even, thus completely solving the problem for these two of the four cases in [72]. Further extending Brian's powerful techniques, one of his students, Mateja Šajna, completed the proof of the conjecture for the remaining two cases with m and n of opposite parity.

Following these break-through results, the focus of research in this area shifted to other problems, including decompositions of complete and nearly complete graphs into cycles of unequal length. To this problem Brian contributed a profound conjecture [Research Problem 3, *Discrete Math.* **36** (1981), 333–334] as early as 1981; namely, that K_n for n odd, and $K_n - I$ for n even, can be decomposed into cycles of lengths $m_1, m_2, \ldots, m_k$ if and only if the sum $m_1 + m_2 + \ldots + m_k$ of the cycle lengths equals the total number of edges. This conjecture remains wide open to this day, although it has been proved in many special cases and no counterexamples have been found.

Recently, working with Heather Gavlas, Mateja Šajna, and Helen Verrall, Brian was able to extend his pioneering techniques for constructing cycle decompositions of complete and nearly complete graphs to complete symmetric digraphs [74], thus proving a conjecture of Bermond and Faber, which had been open since 1976. This conjecture claimed that the complete symmetric digraph on n vertices (with one arc in each direction between each pair of vertices) can be decomposed into directed cycles of length m if and only if the obvious necessary condition that the cycle length m divide the number of arcs $n(n-1)$ is satisfied, except in the case that the pair (m, n) is one of $(3, 6)$, $(4, 4)$, and $(6, 6)$, in which case such a decomposition does not exist. Despite much effort invested into proving this conjecture over the years, only partial results had been known before Brian tackled the problem. The paper [74] received high acclaim, and was for a while featured on the JCTA web page as one of the most downloaded articles.

3.2 The Oberwolfach problem

The Oberwolfach problem asks if it is possible to decompose the complete or nearly complete graph into isomorphic 2-factors, each a disjoint union of cycles of specified lengths. More precisely, $OP(m_1, \ldots, m_k)$ asks if, for $n = m_1 + \ldots + m_k$, it is possible to decompose K_n for n odd, and $K_n - I$ for n even, into isomorphic 2-factors, each consisting of one cycle of each of the lengths $m_1, \ldots, m_k$. If $m_1 = \ldots = m_k = m$, the notation $OP(k; m)$ is used. The problem was first posed by Ringel in 1967 for complete graphs, and in 1979 for nearly complete graphs. Since then, a positive answer has been obtained in many special cases, and a negative answer in a few cases that are widely believed to be the only exceptions, but the complete solution of the problem has been stubbornly alluding the researchers, and the problem seems to be only gaining in notoriety.

Brian's first contribution [38] in this area, with Roland Häggkvist, was the solution of $OP(k; m)$ for all even m. In other words, they established that if m is even and divides n, then $K_n - I$ can be decomposed into 2-factors whose components are cycles of length m. A few years later, Brian teamed up with Paul Schellenberg, Doug Stinson, and David Wagner to attack $OP(k; m)$ with m odd, and in a single paper [43] they solved the problem for all cases but $k = 4$. Thus, they proved that if m is an odd divisor of n and $n \neq 4m$, then K_n for n odd, or $K_n - I$ for $n > 6$ even, can be decomposed into 2-factors whose components are cycles of length m. The case $k = 4$ was settled by Hoffman and Schellenberg a couple of years later [*Discrete Math.* **97** (1991), 243–250]. Brian's work was thus of crucial importance in the complete solution of the Oberwolfach problem for fixed cycle length. He is also the author of a survey paper [67] on the Oberwolfach problem.

3.3 Other work on factorisations and decompositions

With his strong interest in Hamilton cycles as well as cycle decompositions, Brian had a special fondness for hamiltonian decompositions: that is, decompositions into Hamilton cycles and possibly one 1-factor. He showed [24] that a connected vertex-transitive graph of order $2p$ admits a hamiltonian decomposition if $p \equiv 3 \pmod 4$, and with Moshe Rosenfeld [40], he investigated hamiltonian decompositions of graphs arising from simple 4-dimensional polytopes. In particular, they established necessary and sufficient conditions for prisms over simple 3-polytopes to admit a hamiltonian decomposition, and showed that duals of cyclic 4-polytopes always admit such a decomposition. Brian also wrote a survey [47] on hamiltonian decompositions with Jean-Claude Bermond and Dominique Sotteau.

Related to the above is Brian's work on 1-factorisations. With John George [49], he determined some sufficient conditions for the tensor product $X \times Y$ of graphs X and Y to have a 1-factorisation. One of these sufficient conditions is that X be k-regular, and Y have a decomposition into Hamilton cycles or be complete of prime power order k. In an earlier solo paper [34], Brian determined that a line graph of a complete graph admits a 1-factorisation if and only if it has an even number of vertices.

In addition to the papers on path decompositions of directed graphs and the very recent paper on directed cycle decompositions of complete symmetric digraphs, two other early papers of Brian's dealt with decompositions of directed graphs. In [22], written with Kathy Heinrich and Badri Varma, he studied decompositions of the complete directed graph into oriented pentagons, and in [29], certain decompositions into oriented cycles of length one less than the order of the graph; this was joint work with Kathy Heinrich and Moshe Rosenfeld. The decomposition into so-called antidirected cycles, discussed in [29] has the interesting property that for each pair of cycles in the family, there exists an arc that lies in the first cycle while its reversal lies in the second cycle. This is reminiscent of orthogonal factorisations, our next topic.

Let X be a graph, $\mathcal{F} = \{F_1, \ldots, F_k\}$ a factorisation of X, and Y a subgraph of X. Then Y is said to be orthogonal to the factorisation $\mathcal{F}$ if each edge of Y lies in exactly one factor of $\mathcal{F}$. Brian, Guizhen Liu, and Kathy Heinrich studied matchings and $[a, b]$-subgraphs (subgraphs with the degree of each vertex in the interval $[a, b]$) that are orthogonal to certain factorisations in [55]. The same team also wrote a survey [56] on orthogonal factorisations. This survey poses some interesting problems, including the following: for a $2k$-regular graph X and a given 2-factorisation $\mathcal{F}$ of X, is it true that there always exists a matching of X orthogonal to $\mathcal{F}$? Several researchers have become interested in this problem and have shown that the answer is positive if k is sufficiently small relative to the order of the graph. Some asymptotic results are known as well; however, the general problem appears very difficult and remains open to date.

Another result of Brian's, jointly with Joy Morris and V. Vilfred, was the precise characterisation of those values of n for which a self-complementary circulant graph of order n exists, using simple algebraic techniques [69]. This was conjectured in 1963 by Sachs, and was also proven by Frončel, Rosa and Siran using graph-theoretic methods. Several researchers have since begun to study various generalisations of this problem.

4 Hamilton and Other Cycles

The problem of finding Hamilton cycles in graphs is a difficult one that has attracted much interest. Brian has obtained results on this problem for a variety of families of graphs. His most significant contributions in this area have come in two forms: his classification of which generalised Petersen graphs have Hamilton cycles, and his work on finding Hamilton cycles in vertex-transitive graphs. His work on generalised Petersen graphs is very well-known and respected. His work on Hamilton cycles in vertex-transitive graphs was foundational, and his techniques have been widely employed by others. Brian wrote a highly influential survey on the problem of finding Hamilton cycles in vertex-transitive graphs [30].

It has been conjectured that all but a finite number of connected vertex-transitive graphs (the Petersen graph being one exception) have a Hamilton cycle, and that all connected Cayley graphs on at least 3 vertices are hamiltonian. Brian has made some of the most significant progress towards proving these conjectures. Brian's first result in this area was to prove [23] that all connected vertex-transitive graphs on p vertices are not only hamiltonian, but Hamilton-connected: that is, that there is a Hamilton path between any two vertices in such a graph. Using this, he showed that the Petersen graph is the only non-hamiltonian, connected, vertex-transitive graph on $2p$ vertices. This work spurred on a number of other researchers, who were gradually able to prove related results for values such as $4p$ and pq, sometimes obtaining only a Hamilton path, and sometimes requiring the stronger hypothesis that the graph be a Cayley graph in order to guarantee the existence of a Hamilton cycle.

Brian also proved [28,37] that all of the generalised Petersen graphs $\mathrm{GP}(n,k)$ ($k < n/2$) are hamiltonian, except for the case in which they were already known not to be, $\mathrm{GP}(n,2)$ where $n \equiv 5 (\mathrm{mod}\ 6)$. In the first paper, he worked with Peter Robinson and Moshe Rosenfeld to prove that for any n, all but finitely many of the generalised Petersen graphs are hamiltonian. The paper that completed the classification was a solo effort. As mentioned above, this work gained him wide recognition.

In a series of three papers, Brian was able to prove that every connected metacirculant graph for which the cardinality of each block is a prime power has a Hamilton cycle, except the Petersen graph [32,39,42]. The first two papers dealt with blocks of prime cardinality, and were joint work with Tory Parsons; Erich Durnberger joined them on the second paper. The extension to prime powers was written by Brian alone. This is the best result known for this family of graphs.

If X is a graph and $\mathrm{Aut}(X)$ contains a semiregular element α, then the quotient graph X/α has as its vertices the orbits of $\langle\alpha\rangle$; two such vertices are adjacent if and only if there is an edge in X joining a vertex of one corresponding orbit to a vertex in the other corresponding orbit. Brian developed three methods of obtaining a Hamilton cycle in X from a Hamilton cycle in X/α. He used these methods to show that if G is metacyclic (with more than 2 elements), and X is the Cayley graph on G with the standard generating set, then X has a Hamilton cycle [44]. The techniques that he developed were later employed in many of the other results on Hamilton cycles in vertex-transitive graphs. In fact, this notion of the quotient graph that he developed has been used far more broadly, in structural results about vertex-transitive graphs.

In joint work with C.Q. Zhang, Brian proved that every cubic Cayley graph on a dihedral group has a Hamilton cycle [46]. He later extended this — now working with C.C. Chen and Kevin McAvaney [64] — to show that some families of these are in fact Hamilton-laceable (this is the best possible result for bipartite graphs: that there is a Hamilton path connecting any two vertices that are an odd distance apart). Whether or not all Cayley graphs on dihedral groups are hamiltonian remains an open problem, despite the considerable interest that it has attracted.

One of the major results in this area, was the proof [Chen and Quimpo, *Combinatorial mathematics, VIII (Geelong, 1980)*, 23–34, *Lecture Notes in Math., 884, Springer, Berlin*, 1981] that every Cayley graph on an abelian group is Hamilton-laceable if the graph is bipartite, and Hamilton-connected otherwise. Brian and Yusheng Qin were able to extend this result to all hamiltonian groups: that is, finite non-abelian groups in which every subgroup is normal [71]. Brian also suggested another way of determining the prevalence of Hamilton cycles in graphs, by characterising their Hamilton space. The Hamilton space of a graph is the subspace of its cycle space that is generated by its Hamilton cycles. Working with Stephen Locke and Dave Witte, he determined the Hamilton space for any Cayley graph on an abelian group precisely, and showed that it is almost always equal to the cycle space [48].

Brian's results on Hamilton cycles in graphs that are not vertex-transitive include results on hamiltonian properties of matroid base graphs (where the bases of the matroid form the vertices of the graph, with adjacencies where the corresponding bases differ in exactly one element) [45] (with Gui Zhen Liu); and the block-intersection graphs of certain pairwise-balanced designs [50] (with Kathy Heinrich and Bojan Mohar) and of certain balanced incomplete block designs [54] (with Donovan Hare).

With the exception of results dealt with in the section on decompositions, by far Brian's most significant result on cycles that are not Hamilton cycles, was his proof with Luis Goddyn and C.Q. Zhang, that graphs with the circuit cover property are precisely those graphs that have no subgraph homeomorphic to the Petersen graph [60]. A graph has the circuit cover property if for any admissible weight function w on its edges, there exists a list of circuits in the graph, such that each edge e belongs to precisely $w(e)$ of these circuits. A weight function is admissible providing that the obvious necessary conditions hold: the sum of weights of the edges of any edge cut is even and is at least twice the weight of each edge of the cut. This work extended previous results by Seymour and by Brian himself (working with C.Q. Zhang) [58]. It has in particular two nice consequences for bridgeless graphs that do not contain any subgraph homeomorphic to the Petersen graph. Firstly, they satisfy the cycle double cover conjecture, that is, each of them has a family of circuits such that each edge belongs to exactly two of these circuits. Secondly, for these graphs the problem of finding the minimum sum of lengths of circuits for coverings of the edges by circuits is equivalent to the classical Chinese postman problem. Brian and his co-authors' fundamental work on this has led to solutions of several open problems and has important impacts far beyond the graph decomposition area, in such fields as graph embedding, cycle cover optimisation, flow problems and graph colourings.

5 Other Work

Brian's work has spanned a broad variety of research topics, in addition to his main interests. He has published papers on such topics as infinite binary sequences [12] (with Taylor Ollman and Brooks Reid), geometric constructions of graphs [17] (with Moshe Rosenfeld), magic cubes [27] (with Kathy Heinrich), so-called "amida" numbers of graphs [52] (with Zhijian Wang), and characterising graphs with a particular adjacency property [53] (with C.C. Chen and Kathy Heinrich). He has also written papers on Ramsey-type problems [15,35,41], in joint work with various combinations of Tom Brown, Martin Gerson, Geňa Hahn, Kathy Heinrich, and Pavol Hell.

His interest in applications has led him to publish papers relating to other fields. One of his results [16], which appeared in the *Canadian Journal of Chemistry*, applies enumeration techniques to a problem about chemical isomers; it was written with Sam Aronoff. In another paper, he worked with Peter Eades and Gordon Rose on a question that had been posed in *Theoretical Computer Science*, about the number of productions required to produce the language $L_n = \{km : 1 \leq k, m \leq n, k \neq m\}$. An upper bound for this number had been established; Brian and his collaborators produced a lower bound [36].

Probably his most important paper to date, outside of his main research interests, was on matching designs, and was written with Kathy Heinrich. Here, a k-matching is a set of k independent edges in a graph. They considered the problem of constructing (n, k, λ)-matching designs, i.e. collections of k-matchings of K_n for which any pair of independent edges of K_n lies in exactly λ of the k-matchings. This generalised earlier study of hyperfactorisations (the special case where n is even and $k = n/2$). They concentrated on the case $k = 3$ and gave some constructions for the particularly interesting (and difficult) case $\lambda = 1$ [51]. They also considered the bipartite analogue of these matching designs. This paper laid foundations for subsequent constructions of matching designs by other authors.

A recent research area of Brian's [78] has been determining the numbers of pursuers required to find an intruder in a graph, with particular emphasis on Cayley graphs, whose structural properties can make this problem easier to solve. For some years, he led a research project on this topic, partially funded by the Canadian government's Communications Security Establishment. Other project members have included Anthony Bonato, Nancy Clarke, Danny Dyer, Geňa Hahn, Denis Hanson, Jeannette Janssen, Xiangwen Li, Richard Nowakowski and Boting Yang. Brian's interest in this topic stems from an old paper by his former collaborator, Tory Parsons, who had been trying to optimise searching patterns for a lost spelunker in a cave system.

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