

Axioms for the set of numbers $S = \mathbb{Q}$ or $S = \mathbb{R}$

For every $x, y, z \in S$:

A1) $x + y \in S$	M1) $x \cdot y \in S$	(closure)
A2) $(x + y) + z = x + (y + z)$	M2) $(x \cdot y) \cdot z = x \cdot (y \cdot z)$	(associativity)
A3) $x + y = y + x$	M3) $x \cdot y = y \cdot x$	(commutativity)
A4) $x + 0 = x$	M4) $x \cdot 1 = x$	(identity)
A5) Given x , there is $(-x) \in S$ s.t.	M5) Given x , there is $\frac{1}{x} \in S$ s.t.	
$x + (-x) = 0$	$x \cdot \frac{1}{x} = 1$	(inverse)
DL) $x \cdot (y + z) = x \cdot y + x \cdot z$		(distributivity)

Note: Axioms A1–A5 are also satisfied by set $S = \mathbb{Z}$.

Another note: When writing multiplication, often we omit to write the symbol \cdot .

Power of ...

By definition, we write for positive integer n :

$$x^n = \underbrace{x \cdot x \cdot \dots \cdot x}_{n \text{ times}}$$

Very often the value of n is 2. In this case, we call $y = x^2$ the square of x . We have another notation for the inverse of the square of x , $\sqrt{y} = x$. Remember that \sqrt{y} is defined only if y is positive (if we restrict our discussion to set \mathbb{R}).

As a consequence of these notations, we have the following rules:

$$\begin{aligned} x^n \cdot x^m &= x^{n+m} \\ (x^n)^m &= x^{n \cdot m} \\ \sqrt{y} &= y^{\frac{1}{2}} \end{aligned}$$