Math 2865 Assignment #2 Due: Oct 18th, 2007

- 1) How many permutations of the letters *a*, *b*, *c*, *d*, *e*, *f*, *g* contain neither the pattern *bge* nor the pattern *eaf*?
- 2) How many permutations of the letters *a*, *b*, *c*, *d*, *e*, *f*, *g* have either two or three letters between *a* and *b*?
- 3) A woman has 9 close friends.
 - a) In how many ways can she invite 6 of these to dinner?
 - b) Repeat a) if two of her friends are divorced (from each other) and don't want to attend together.
 - c) Repeat a) if the friends consist of three single people and three married couples and, if a husband or wife is invited, the spouse must be invited too.
- 4) How many different collections of ten coins can be made from pennies, nickels, dimes, and quarters?
- 5) What is the coefficient of x²⁵ in the binomial expansion of $\left(2x \frac{3}{x^2}\right)^{58}$?
- 6) How many binary sequences of length 20 are there that start with a run of 0s, that is, a consecutive sequence of at least one 0; then a run of 1s; then a run of 0s; then a run of 1s; and finally finishing with a run of 0s.
- 7) Show by a combinatorial argument that $\binom{2n}{2} = 2\binom{n}{2} + n^2$ Hint: Look at it as

selecting 2 people out of n men and n women. How many ways can you do that?

8) Show that $\binom{n}{1} + 6\binom{n}{2} + 6\binom{n}{3} = n^3$