Mathematics 2865 Assignment #2
Solutions

1) Find the number of permutations that contain either *bge*, *eaf* or both and subtract from the total number of permutations (7!). Let A be the set of permutations which contain the pattern *bge*: |A| = 5! (thinking of *bge* as a single symbol). Similarly, if B is the set of permutations which contain *eaf*, |B| = 5!. For permutations to contain both *bge* and *eaf* it must contain *bgeaf*, so $|A \cap B| = 3!$ The number containing either *bge* or *eaf* is $|A \cup B| = |A| + |B| - |A \cap B| = 5! + 5! - 3! = 234$. Hence, the answer is 7! – 234 = 5040 – 234 = 4806
2) Use addition rule. First assume there are two letters between *a* and *b*. The number of permutations here is 2*P(5,2)*4! = 960 (there are P(5,2) permutations of 2 letters that don't include *a* and *b*, then 2 ways to place *a* and *b* on each end and finally glue those 4 letters together and arrange with the rest of the letters in 4! ways). Next assume there are three letters between *a* and *b*. The number of such permutations is 2*P(5,3)*3! = 720. By the addition rule, the answer is 960 + 720 = 1680
3) Order doesn't matter here!

   a) $\binom{9}{6} = 84$

   b) The number of groups that include divorced couple is $\binom{7}{4}$ (couple is already invited, so choose the other 4 guests out of the remaining 7). Thus, the number in which not both of these people are present is $\binom{9}{6} - \binom{7}{4} = 84 - 35 = 49$

   c) Our hostess must either invite two married couples and two singles or all three married couples. If she invites two married couples and two singles the answer is $\binom{3}{2}\binom{3}{2}$ and if she invites all three married couples the answer is 1 since there is only one way to choose 3 couples out of 3. Now use addition rule, $\binom{3}{2}\binom{3}{2} + 1 = 10$

4) Think of the coins as being indistinguishable object. Now place them into distinguishable bins named pennies, nickels, dimes and quarters. There are $\binom{10+4-1}{10}$ ways to do that, or 286.
5) The general term is

$$\binom{58}{k}\left(\frac{-3}{x^2}\right)^{58-k}(2x)^k = \binom{58}{k}(-3)^{58-k}(x^{-2})^{58-k}2^k x^k = \binom{58}{k}(-3)^{58-k}2^k x^{-116+3k}$$

Now we want the coefficient of x^{25}, therefore -116+3k must equal 25. It follows that $k = 47$. Plug in the value for k in the coefficient above and the answer is $\binom{58}{47}(-3)^{11}2^{47}$

6) 0 _____ 1 _____ 0 _____ 1 _____ 0 _____ this is the sequence. Here we have 5 bins and 20 – 5 objects to distribute. Consider these 15 0s and 1s as indistinguishable objects being placed in distinguishable bins. Therefore, there are $\binom{15+5-1}{15}$ = 3876 such sequences.

7) $\binom{2n}{2} = 2\binom{n}{2} + n^2$ Left hand side: Selecting 2 people out of n men and n women. There is $\binom{2n}{2}$ ways to do that. Right hand side: Same selection in a different way. We can choose two men, or two women or one of each. If it's men, then number of ways is $\binom{n}{2}$, if it's women the number of ways is $\binom{n}{2}$ and if it's one of each then the number of ways is $n*n$, since there are n choices for men and n for women. Use addition rule and the result follows.

8) $\binom{n}{1} + 6\binom{n}{2} + 6\binom{n}{3} = \frac{n!}{1!(n-1)!} + 6\frac{n!}{2!(n-2)!} + 6\frac{n!}{3!(n-3)!}$

$= n + 3n(n-1) + n(n-1)(n-2) = n + 3n^2 - 3n + n^3 - 3n^2 + 2n = n^3$