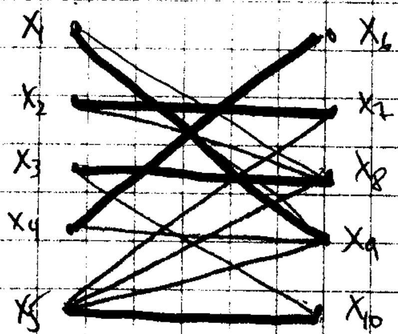
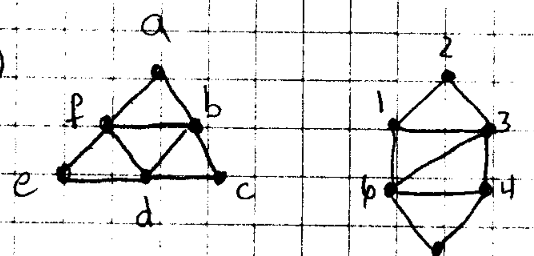
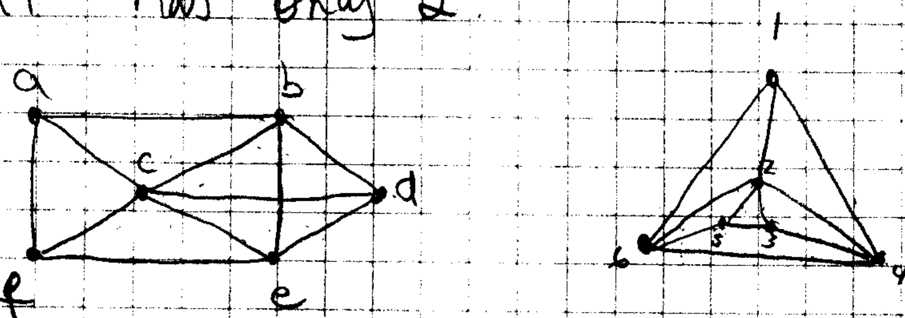
Math 2865 Solutions to Assignment #4

1) 

This matching is maximal because M-alternating path does not exist. All vertices on the left and on the right are a part of the matching, so any Alternating path would not be maximum.

2) a) 

These two are not isomorphic, the graph on left has 3 vertices of degree 2, but the graph on the right has only 2.

b) 

These 2 are isomorphic, check degrees of vertices, subgraphs

Isomorphism: $c \to 2$ $\quad f \to 3$

$b \to 6$

$e \to 4$

$d \to 1$

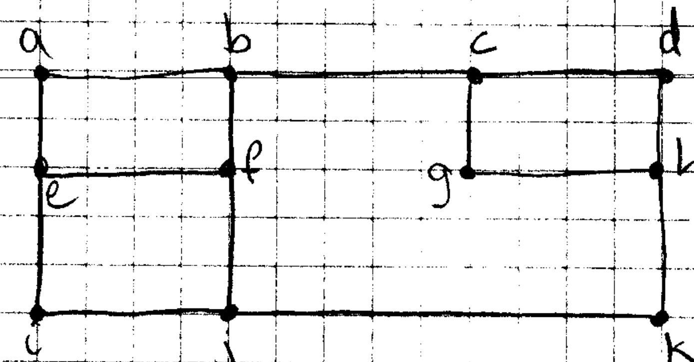
$a \to 5$

c) Not isomorphic. On the right graph, there is a triangle with vertices of degrees 5-3-3 There is no such triangle on the right

3) For an Euler cycle to exist we need all vertices to be even. In K_n, that happens when n is odd. For K_n to have Euler trail but no Euler cycle, we need two odd degree vertices. That only happens when $n=2$ $K_2 =$ •—• trail, but no cycle

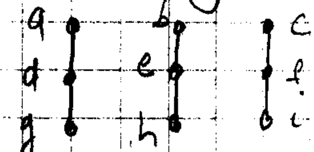
For $K_{r,s}$ to have an Euler cycle, all vertices have to have even degrees, so s and r have to be even.

4)



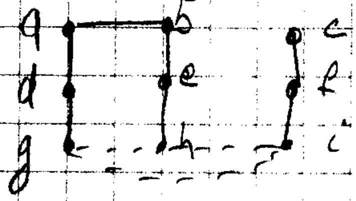
There are 6 vertices of odd degree, so an Euler set has at least 3 trails. You need to lift your pencil when you stop one trail and start a new one - lift twice at least

#5 Apply the rule for degree 2 vertices (d, e, f) to force



to be on cycle.

Assume ab is on cycle. Then neither ac nor bc can be by another rule. So delete them giving



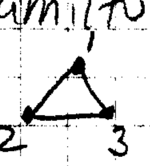
This forces c to be an endpoint of a path so there is no Hamilton cycle.

To get a Hamilton path use hi or gi.

#6 a) If $n=2$, there are no cycles so assume $n \geq 3$. Numbering the vertices $1, 2, \cdots, n$ and beginning at 1, there are $n-1$ choices for for the second vertex, then $n-2$ choices for the third vertex and so on. Altogether, there are $(n-1)!$ different Hamiltonian cycles in K_n.

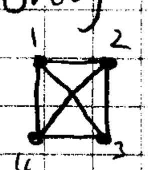
b) If $n=2$, there are no Hamiltonian cycles.

If $n=3$, the graph is



, 123 and 132 are the only Hamiltonian cycles

If $n=4$



The Hamiltonian cycles are 1234, 1243, 1324, 1342, 1423, 1432.