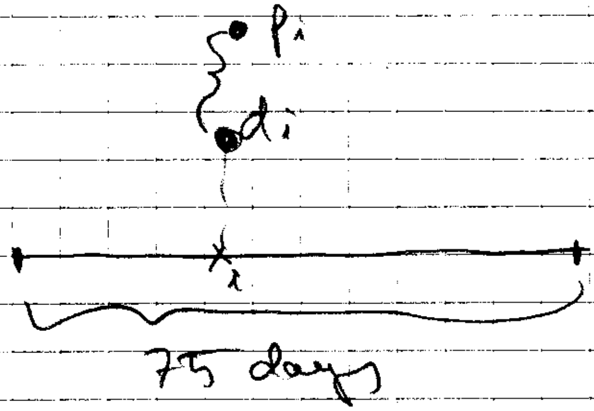
(1).



let d_i = # of games played from day 1 to day i

let $p_i = d_i + 24$.

- we get 2 sets of points with coordinates (i, d_i) and (i, p_i) for $i \in \{1, .., 75\}$, a total of 150 points $(2 \cdot 75)$

- $p_i \leq 125 + 24 = 149$
- Since there is at least a match every day, no 2 points from same series (i, d_i) or (i, p_i) can have same y-coord., but by p-h-p there must be 2 pts (from different series) whose y-coordinate are the same. let them be p_i & d_j This means that the difference in y-coord (# of games played) between the 2 points (i, d_i) and (j, d_j) is exactly 24. qed

b) If we use 5, the max y-coordinate is $125+5$ & p-h-p can be applied to prove that at least 2 days exist s.t. there are exactly 5 games are played between those 2 days.

However, for value 25, the p-h-p cannot be applied & it is possible to have no 2 days s.t. the # of games played in that interval is = 25.

For ex:

# games	1	1		1	25	1	- -	1	25			
day	1	2	.	24	25	26	- -	49	50	- - -	etc	

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(2) a) For the first letter in string, we have 26 choices
2nd letter, also 26 choices - -

$\Rightarrow 26^6$ - # of different strings

b) There are $P(25,5) = \frac{25!}{20!}$ different 5 letter strings with distinct letters, not including 'a'.

6 positions for letter 'a'

$\Rightarrow$ answer $= 6 \cdot \frac{25!}{20!}$

2. c) Similar to b.

We have 2 groups $<\begin{matrix} ab \\ ba \end{matrix}$ & 24 candidate letters for 4 letter strings.

$P(24,4) = \frac{24!}{20!}$ such 4 letter strings &

5 positions to place ab within the string.

The same # of strings if we place ba.

$\Rightarrow$ answer $= \boxed{10 \cdot \frac{24!}{20!}}$

d) Again, there are $P(24,4)$ different 4 letter strings but now, among the 6 different positions for a & b we choose 2.

This is done in $\binom{6}{2}$ ways.

For each choice, there is only one way to place a & b in these positions & we put the $P(24,4)$ different strings on the other ones.

Total $\binom{6}{2} P(24,4)$.

3. a) There is no restriction how the rings are selected, so this can be done in $\binom{25}{5}$ ways (order not important)

b) For left, she chooses 3 out of 8 silver and 2 out of 12 gold, in $\binom{8}{3}\cdot\binom{12}{2}$ ways.

There are 20 rings left, so for right hand she picks rings in $\binom{20}{5}$ different ways.

All these operations are performed in order to complete a configuration, so we multiply the results:

$$\boxed{\binom{8}{3}\binom{12}{2}\binom{20}{5}}$$

3.c) No 2 rings on a finger are of the same material.

Middle finger : choose 1 ring from 8 silver,
1 ring 12 gold
1 — " — 5 titanium

So there are $8 \cdot 12 \cdot 5$ choices.

Index finger : three cases silver + gold (α)
silver + titanium (β)
gold + titanium (γ)

(α) : $7 \cdot 11$ choices (some rings already taken for middle finger)

(β) : $7 \cdot 4$

(γ) : $11 \cdot 4$

Total is $7 \cdot 11 + 7 \cdot 4 + 11 \cdot 4$ because she has one choice out of the 3 cases α, β, γ.

Ruby : only 1 ring to choose, so $25 - 3 - 2 = 20$ possibilities

Answer is $8 \cdot 12 \cdot 5 \cdot (7 \cdot 11 + 7 \cdot 4 + 11 \cdot 4) \cdot 20 =$

$= 480 \cdot 149 \cdot 20 = 480 \cdot 2980 = 1,430,400$

(you don't need to calculate)

$$\begin{array}{r} 2980 \\ 480 \\ \hline 238400 \end{array}$$

3.d) The only difference with 3.c) is that order is important. So, @ each finger, once the set of rings are chosen, there are additional 3! (for middle) & 2! (for index) possibilities).
So, take the answer from c) and multiply by $3! \cdot 2!$.
Answer $\underbrace{1430400 \cdot 12}$.

(4) a) $A \subseteq B$ so $A \cap B = A$ & $A \cap B \cap C = B \cap C$

$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C| =$$
$$= 12 + 18 + 20 - 12 \cancel{-6} - 5 \cancel{+6} =$$
$$= 50 - 17 = 33.$$

b) $A \cap B = \emptyset$ so $A \cap B \cap C = \emptyset$

$$\Rightarrow |A \cup B \cup C| = 12 + 18 + 20 - 0 - 2 - 11 + 0 =$$
$$= 50 - 13 = 37$$

(5) characteristic equation is:

$$x^2 - 5x + 6 = 0$$

$$x_{1,2} = \frac{5 \pm \sqrt{25-24}}{2} = \begin{cases} 3 \\ 2 \end{cases}$$

So, $a_n = c_1 2^n + c_2 3^n$

$a_0 = c_1 + c_2 = 1 \quad (1)$

$a_1 = 2c_1 + 3c_2 = 0 \Rightarrow 2c_1 = -3c_2$

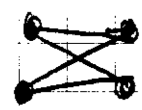
From (1) $\Rightarrow 2c_1 + 2c_2 = -3c_2 + 2c_2 = 2$

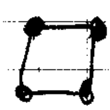
$$\begin{cases} c_2 = -2 \\ c_1 = 3 \end{cases}$$

So, $\boxed{a_n = 3 \cdot 2^n - 2 \cdot 3^n}$

6. a) Yes. A correct mapping from the vertices of the graph on the left with the vertices of the graph on the right is like follows:
- map the deg. 2 vertex to the deg 2 vertex.
- map the neighbours of the deg 2 vertex to the neighbours of deg 2 vertex
- map the other 2 vertices in any way, as long as we have a bijection.

b) Yes, K_4 is an induced subgraph, and so is K_3.

K_{22} is  which is isomorphic to a C_4 (cycle of 4 vertices)



However, any cycle of 4 vertices in the graph has an extra edge in the graph, so K_{22} is not an induced subgraph.

(7) No Eulerian circuit (c has degree 3)
No Hamiltonian cycle (d must be visited twice)
Hamiltonian path: f e d c b a i h g

There are only 2 vertices with odd degree
(b & c). So an Eulerian trail exists between
them: b a d e f d g h i a h d c h b c.

(8) (a) Max flow is 8. A cut with capacity 8 is
$\{(ad), (cd), (hd), (gd)\}$

The flow is $f(ad) = 5 = f(de) = f(ef)$
$f(ab) = f(ah) = f(ai) = f(ih) =$
$= f(bh) = f(hc) = f(hd) = f(hg) =$
$= f(gd) = f(cd) = 1$
$f(df) = 3.$ The flow is not unique!

(b) The shortest path is a h d , length 2 .
Second shortest is a h c d, length 3.
not unique!

Important is that the length is 3.