Test 3 solutions

(1) $a_n - \frac{1}{4} a_{n-2} = 0$

$$\left.\begin{aligned} &x^2 - \frac{1}{4} = 0 \\ &x_{1,2} = \pm \frac{1}{2} \end{aligned}\right\} \text{characteristic equation}$$

General solution is of the form

$$a_n = c_1 \cdot \left(-\frac{1}{2}\right)^n + c_2 \left(\frac{1}{2}\right)^n$$

Using the initial conditions:

$$\begin{cases} a_0 = 1 = c_1 + c_2 & (1) \\ a_1 = 0 = -\frac{c_1}{2} + \frac{c_2}{2} \Rightarrow c_1 = c_2 \end{cases}$$

Substituting in (1),

$$1 = 2c_1 \quad , \quad c_1 = \frac{1}{2} = c_2$$

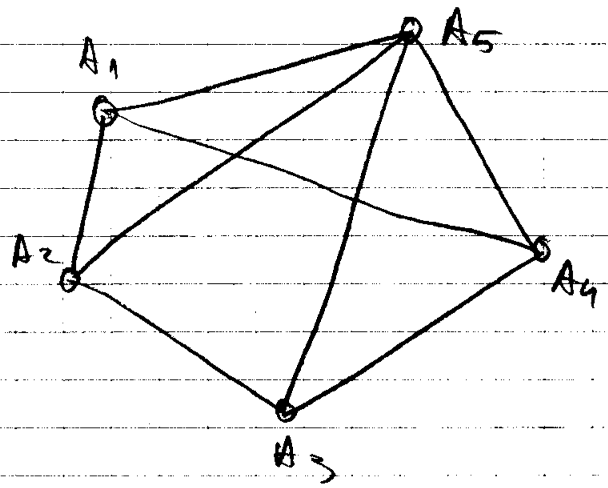
$$a_n = \frac{1}{2} \cdot \left(-\frac{1}{2}\right)^n + \frac{1}{2}\left(\frac{1}{2}\right)^n =$$

$$= -\left(-\frac{1}{2}\right)\left(-\frac{1}{2}\right)^n + \left(\frac{1}{2}\right)^{n+1} =$$

$$= \left(\frac{1}{2}\right)^{n+1} - \left(-\frac{1}{2}\right)^{n+1} = \frac{1-(-1)^{n+1}}{2^{n+1}}$$

(2)

(a)



(b) Cycle with 3? Yes $A_1 A_2 A_5$

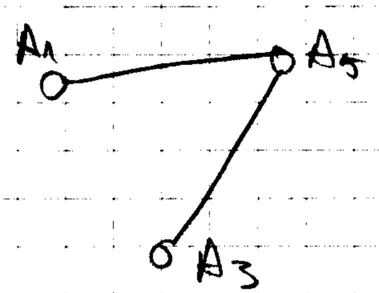
4? Yes $A_1 A_2 A_3 A_4$

5? Yes $A_1 A_2 A_3 A_4 A_5$ (Hamilton cycle)

(c) $\{(A_1 A_2), (A_3 A_4)\}$ is a matching.

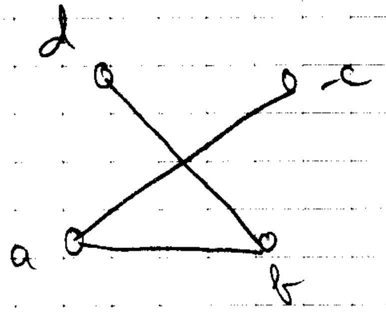
It is also a maximum matching because the graph has 5 vertices so at most 2 edges can be in any matching (a matching with 3 edges requires 6 distinct vertices).

(d)



3. See definitions in textbook

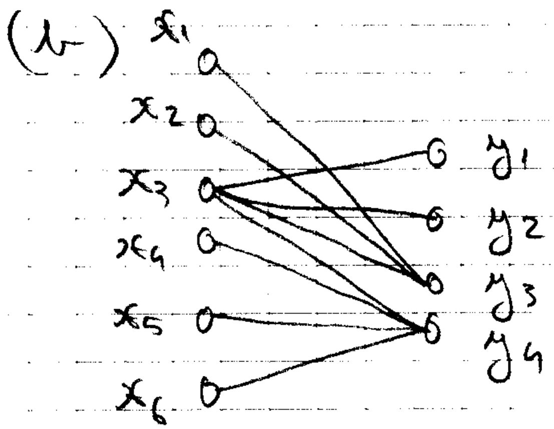
4. (a)



← $\bar{G}$ is the complement of G. Notice that $\bar{G}$ is also a path, thus it is isomorphic to G (which is a path).

One isomorphism is $\{a \to d,\ d \to b,\ c \to a,\ b \to c\}$.
another is $\{a \to c,\ d \to a,\ c \to b,\ b \to d\}$

(b)



A matching of size 3 $M = \{(x_1 y_3), (x_3 y_1), (x_4 y_4)\}$ is maximum.

Each of these 3 edges has an endpoint of degree 1, thus none of these edges may be part of an M-alternating path. For an M-alternating path to exist an edge in matching has to have both endpoints with deg at least 2 because the matching edge is interior to the path.