Due date:
You have until Dec. 4/07 to complete this take home test. Please hand in your answers at the beginning of the lecture.
Marks:
There are 4 problems worth 10 marks each for a total of 40 marks. Please attempt ALL problems.

(10pts) Problem 1: Solve the following recurrence relation with the given initial conditions:

$$a_n = \frac{a_{n-2}}{4},\ n \geq 2,$$
$$a_0 = 1,$$
$$a_1 = 0.$$

(10pts) Problem 2: The *intersection graph* of a collection of sets $A_1, A_2, \ldots A_n$ is the graph that has a vertex for each set and has an edge connecting the vertices representing two sets if these two sets have a non-empty intersection. (EX: for sets $\{1, 2\}$ and $\{2, 3\}$, the graph has 2 vertices connected by an edge)

Given the following collection of 5 sets:

$$A_1 = \{0, 2, 4, 6, 8\} \qquad A_2 = \{0, 1, 2, 3, 4\}$$
$$A_3 = \{1, 3, 5, 7, 9\} \qquad A_4 = \{5, 6, 7, 8, 9\}$$
$$A_5 = \{0, 1, 8, 9\}.$$

(a) (3pts) Draw the intersection graph of the collection of sets. Make sure you **label** the vertices.

(b) (3pts) Does this graph contain cycles with 3, 4, or 5 vertices? For each case, either give a cycle or explain why no such cycle exists.

(c) (2pts) Find a matching of *any* size in this graph.

(d) (2pts) Draw the subgraph induced by the vertices corresponding to sets $\{A_1, A_3, A_5\}$.

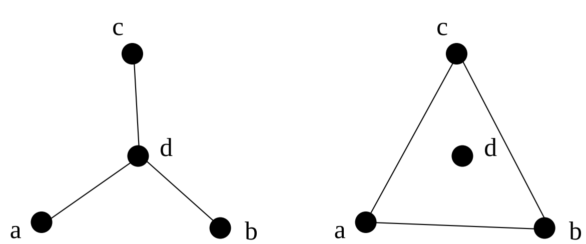
(10pts) Problem 3: Give the definitions for the following terms:

(a) (5pts) What is a bipartite graph?

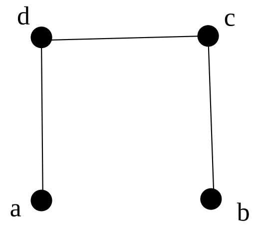
(b) (5pts) When two graphs are isomorphic?

(10pts) Problem 4:

(a) (5pts) The complement of a graph $G = (V, E)$ is denoted by $\overline{G}$ and represents the graph on the same set of vertices like G, but having an edge (u, v) if and only if (u, v) is NOT an edge in G. For example, the following two graphs are complements of each other,



A graph G is called *self-complementary* if G and $\overline{G}$ are isomorphic. Show that the graph drawn below is self-complementary.



(b) (5pts) Find a maximum matching in the graph drawn below and explain why it is maximal.