CPSC 3750 – A.I.	**Due Mar. 31** (in class)
Assignment 4	Total marks: 50

1) **(10 pts)** Using resolution, prove that sentence d is true from the following sentences (implication has the lowest priority):

$$a \Rightarrow (c \Rightarrow b)$$
$$b \vee c \Rightarrow d$$
$$d \Rightarrow f$$
$$\neg a \Rightarrow \neg b \wedge c.$$

Solution:
This problem contains an unintentional error. d cannot be proven true. However, you will get full marks if you correctly follow the steps involving proof with resolution.
To succeed in proving d, the first sentence must read $a \Rightarrow c$. Notice that the sentences are very closely related to the solution of Question 2, with one other unintentional mismatch in the last sentence.
Anyway, first we rewrite the sentences in CNF, replacing $\alpha \Rightarrow \beta$ with $\neg\alpha \vee \beta$, using distributivity of $\vee$ over $\wedge$ and of $\wedge$ over $\vee$, and De Morgan rules. We get in the same order as for the original sentences the following (everything is connected by $\wedge$).

$$\neg a \vee \neg c \vee b$$
$$(\neg b \vee d) \wedge (\neg c \vee d)$$
$$\neg d \vee f$$
$$(a \vee \neg b) \wedge (a \vee c)$$

To attempt a proof for d, we add $\neg d$ to the clauses. The shortest clauses we can entail is a after we get $a \vee d$ (from 2 and 4) and then we resolve with $\neg d$.
Had the sentences been correct, we would have had:

$$\neg a \vee c$$
$$(\neg b \vee d) \wedge (\neg c \vee d)$$
$$\neg d \vee f$$
$$(a \vee \neg c) \wedge (a \vee b).$$

From 1 and 4 we get $c \vee b$ which is resolved with 2 to get $c \vee d$, which then is resolved with the other clause in 2 to get d which gives the empty clause when resolved with $\neg d$. Compare this with your reasoning in Question 2.b.

2) The following database is given, in plain English.

> "If the unicorn is mythical, then it is immortal. but if it is not mythical, then it is a mortal mammal. If the unicorn is either immortal or a mammal, then it is horned. The unicorn is magical if it is horned."

(a) **(5 pts)** Express this database in *propositional logic*. Define your symbols.

Solution:

With:		
	a	unicorn is mythical
	b	unicorn is mamal
	c	unicorn is immortal
	d	unicorn is horned
	f	unicorn is magical

we get:

$$a \Rightarrow c$$
$$\neg a \Rightarrow \neg c \wedge b$$
$$b \vee c \Rightarrow d$$
$$d \Rightarrow f$$

(b) **(15 pts)** Can you prove that the unicorn is mythical? How about magical, horned, and mammal? Explain your proof in plain English.

Solution:

We can only prove the unicorn is horned and magical. Think of 2 cases, mythical or not. In both cases you have that the unicorn ios either immortal or a mammal which is a necessary condition for horned.

3) **(15 pts)** Express the following English sentences in *first order logic*. Define your vocabulary and symbols.

(a) Some student took both Greek and French classes.

(b) Every student that takes French passes it.

(c) The best score obtained in the Greek class is higher than the best score obtained in the French class. Note: You cannot define and use a function such as *best_score(Greek)* because its answer depends on the scores of the students enrolled in *Greek* and not only on the object *Greek*.

Solution:

The solution is not unique. Here is a simple one that is not the most powerful. Let $G(x)$ and $F(x)$ state that x took Greek and French classes respectively. Let $GS(x)$ and $FS(x)$ state that x is a score in the Greek and French classes respectively. Let $PF(x)$ state that x passes French.

$$\exists x\ G(x) \wedge F(x)$$
$$\forall x\ F(x) \Rightarrow PF(x)$$
$$\exists g_{max}\ \exists f_{max}\ GS(g_{max}) \wedge GF(f_{max}) \wedge \big(\forall s_g\ GS(s_g) \Rightarrow g_{max} \geq s_g\big) \wedge \big(\forall s_f\ GF(s_f) \Rightarrow f_{max} \geq s_f\big) \wedge g_{max} > f_{max}.$$

4) **(5 pts)** Is the following sentence valid? Explain your answer.

$$\exists x\ \exists y\ x = y.$$

Solution:

As long as the universe we try to describe contains at least one object, the sentence is true (both x and y would point to the same object).