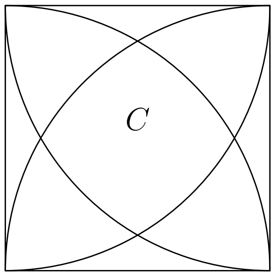
CPSC 4850/5850 – Algorithms in OR	
Assignment 1, total xx pts	**Due XXX**

Instructions: hand in your written or typed answers in class on the due date shown. If you are taking the class as undergraduate, problems marked *(GRAD)* are optional.

Problem 1) Given a square in two dimensional Euclidean space with sides equal to 1, four circular arcs with the centers in the corners of the rectangle are drawn as in the figure. Compute the area of the central region marked C by formulating a system of three linearly independent equations in three unknowns. Use your knowledge about the properties of circles and triangles. Briefly explain your formulation.



Problem 2) A refinery makes three types of gasoline, regular, bronze, and gold, by blending three types of crude oil distillates, A, B, and C. Formulate an LP (linear program) to determine the blending plan that maximizes profit, given the data below. Define your variables and explain the constraints.

Distillate	OC.R	Availability (brl/day)	Cost ($/brl)
A	83	20,000	26
B	88	25,000	30
C	93	15,000	34

Gasoline	Min Oc.R	Max % A	Min % C	Selling price ($/brl)
Regular	87		20	33
Bronze	89	15	30	41
Gold	90	60	40	48

Assumptions: The law of conservation of mass (or volume) of ingredients: blending α litres of A and β litres of B gives $(\alpha + \beta)$ litres of fuel. Proportionality of octane rating; blending α litres of A with β litres of B gives fuel with octane rating $\dfrac{83\alpha + 88\beta}{\alpha + \beta}$

Problem 3) [**pts**] Given is the following primal LP, which we denote by P. Pay attention to the objective, as P is different from the example seen in class.

$$
\begin{aligned}
\min x_1 + x_2 + 2x_3 \\
x_1 + x_2 \leq 3 \\
x_1 + x_3 \leq 4 \\
x_1 + 2x_2 - 7x_3 \leq 1 \\
x_2 + 3x_3 \leq 8 \\
x_i \geq 0,\ i \in \{1, 2, 3\}.
\end{aligned}
$$

(a) Transform P into standard form, P_{std}. Explain your notation.

(b) Let D and D_{std} be the duals of P and P_{std} respectively. Define D and D_{std}, explain your notation, and briefly argue the correctness of your derivations. What is the relationship between D and D_{std}?

(c) Write the dual of D. Let this dual be D'. What is the relationship between P and D'?

Problem 4) [**pts**] (GRAD) Write the dual of the following LP. Explain your notation and your derivation.

$$\min x_1 + x_2 + x_3$$
$$x_1 + x_2 \leq 3$$
$$3x_1 - 2x_3 \geq -4$$
$$2x_1 - x_2 + 5x_3 = 1$$
$$x_i \geq 0,\ i \in \{1, 2, 3\}.$$

Problem 5) [**pts**] (GRAD) Write the dual of the following LP. Explain your notation and your derivation. Notice that the decision variables are NOT constrained to be positive!

$$\min 2x_1 + 3x_2 + x_3 - x_4 - 2x_5$$
$$x_1 + x_2 + x_4 + x_5 = 4$$
$$x_2 + 2x_3 + 3x_4 - x_5 = 7$$

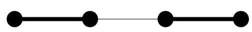
Problem 6) [**pts**] (GRAD) Consider the following LP which attempts to model the problem of selecting a minimum spanning tree of a connected edge weighted graph. Let $G = (V, E)$ be the graph with $w : E \to \mathbb{R}$ as edge weights, and x_e be the decision variable assigned to every edge $e \in E$ with the convention that

$$x_e = \begin{cases} 1, & e \text{ is in the spanning tree} \\ 0, & \text{otherwise} \end{cases}$$

The LP formulation is

$$\min \sum_{e \in E} w(e) x_e,$$
$$\sum_{e \in \delta(v)} x_e \geq 1, \qquad \forall v \in V,$$
$$x_e \geq 0,$$

where $\delta(v)$ is the set of edges incident to vertex v. The constraint in the LP says that every vertex v in the graph must be spanned by at least one edge from the spanning tree. However, this formulation on the simple path graph in the figure returns a set of edges that does not form a spanning tree. Write a *set of* constraints that would correct the LP by disallowing such spanning forests on any graph. The number of constraints in the set may be exponential.



Problem 7)