Welcome to CPSC 4850/5850 - OR Algorithms

Course Outline

- Instructor: Robert Benkoczi, D520.
- Resources:
  1. Text: Operations Research (95), Katta Murty; highly recommended.
  2. Papers on course web page.
  3. Other links will be posted as necessary.
  4. Tutorials & Octave manual (programming).
- Grading:
  - Assignments: problem solving and coding.
  - Paper presentation. A schedule of paper presentations will be posted on the course web page.
  - Project: 3 weeks (view as a larger assignment); topics will be provided, but free to chose your own. Consult your instructor.
  - Take home exam (24 or 48 h).

Course Outline

GRADUATE VS UNDERGRADUATE WORK:

- Lectures: same.
- Assignments: extra questions for graduate students.
- Paper presentation: if you are undergraduate, guidance will be provided; talk to your instructor before choosing paper.
- Project: if you are undergraduate, consult your instructor before choosing project.
- Take home exam: extra questions for graduate students; same time for completing questions for both graduate & undergraduates.

What is OR?

OR = techniques for optimizing the performance of systems.

Example

1. Product mix problems: planning manufacturing process (maximize profit).
2. Machine scheduling (minimize completion time).
3. Matching problem (minimize cost of matching).
4. Generalized assignment problem: agents and tasks, cost & profit for tasks, budget for agents (maximize total profit)
5. Shortest paths in a weighted graph
6. Protein-lattice alignment (minimize total error).
7. Scheduling for courier company: assign people, vehicles, routes and jobs (maximize profit subject to capacity constraints)
8. ...

Approaches to solving optimization pb.

1. Using combinatorics
2. Using numerical variables and algebra/calculus (mathematical programming).

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Product mix (numerical by nature):

Profits and requirements for products A,B

	A	B	Availability
RM1	2	1	15
RM2	1	1	12
RM3	1	0	5
profit	15	10	

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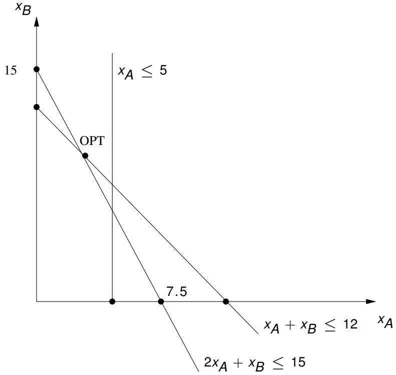
COMBINATORIAL ALGORITHM:

- maximize production for A (largest profit)
- produce B if materials left.

Solution: 5 A and 5 B. Profit: 125.

Approaches to solving optimization pb.



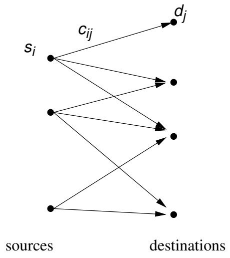
x_A, x_B: production (decision variables)

$$
\begin{aligned}
2x_A + x_B &\leq 15 \\
x_A + x_B &\leq 12 \\
x_A &\leq 5
\end{aligned}
$$

Max $15x_A + 10x_B$ (objective function)

OPT profit: 135 at $(3, 9)$

Other examples. Transportation



Given

Supply s_i for source i;
Demand t_j for target j;
Transportation cost c_{ij};

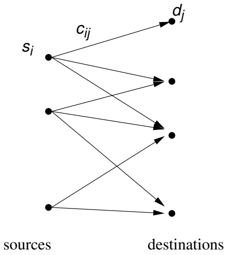
Output

Shipment x_{ij} from source i to target j

Objective

Minimize transportation cost

Other examples. Transportation



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Balanced transportation

Total supply = total demand

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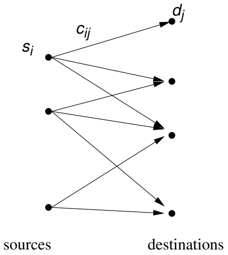

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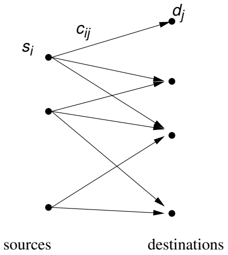

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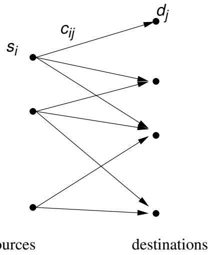

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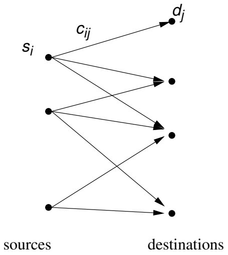

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Decision variables

x_{ij}: shipment from i to j.

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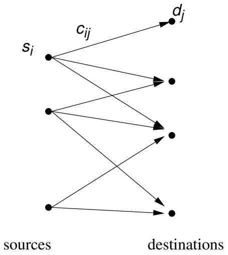

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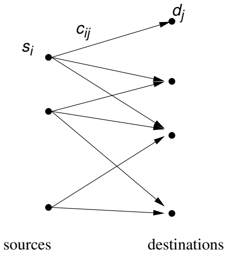
$$\sum_j x_{ij} = s_i, \quad \forall i$$

$$\sum_i x_{ij} = t_j, \quad \forall j$$

$$x_{ij} \geq 0.$$

Other examples. Transportation



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$$\min \sum_i \sum_j c_{ij} x_{ij}$$

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- Computing the optimal solution.
- Values of *slack* variables at optimal solution: critical resource analysis.
- *Marginal cost* analysis: how to change supplies or requirements to improve the objective.

Slack variables at OPT

Recall product mix problem:

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\begin{aligned}
\max\ 15x_A + 10x_B \\
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Recall product mix problem:

$$\max 15x_A + 10x_B$$
$$2x_A + x_B \leq 15$$
$$x_A + x_B \leq 12$$
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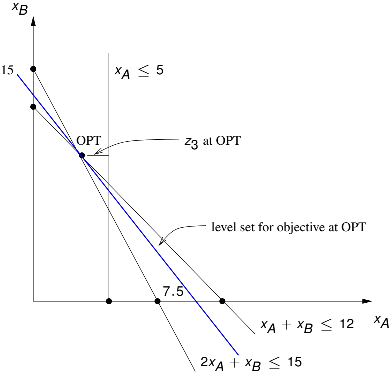
We can translate inequalities in equalities using slack variables (useful for LP algorithms like simplex).

$$2x_A + x_B + z_1 = 15$$
$$x_A + x_B + z_2 = 12$$
$$x_A + z_3 = 5$$
$$z_1, z_2, z_2 \geq 0$$

Slack variables at OPT



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z_1, z_2, z_2 &\geq 0
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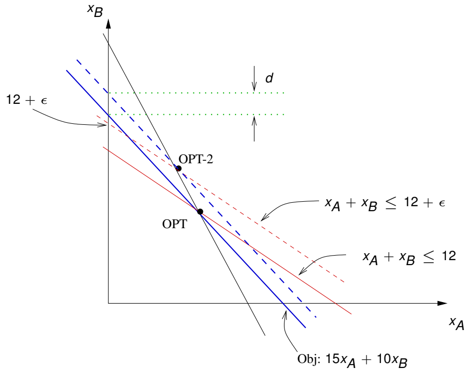
OPT value: 135 at $(3, 9)$
$z_3 = 2, \; z_1 = z_2 = 0$
(slack variables show critical resources)

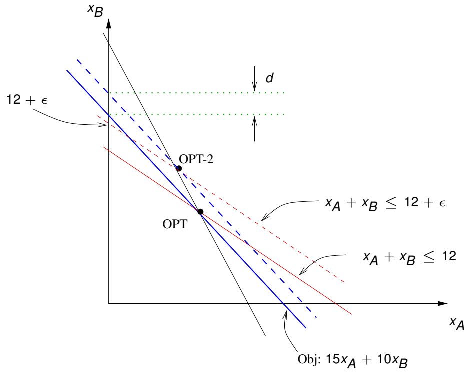
Marginal values



$$\max \; 15x + 10x_B$$
$$2x_A + x_B \leq 15$$
$$x_A + x_B \leq 12 + \epsilon$$
$$x_A \leq 5$$

Marginal value of constraint (2) is $\pi_2 = 10d$.

Marginal values



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Definition

Marginal value of constraint (i) = change in optimal cost if RHS of (i) is perturbed by 1.

Marginal values

Profits and requirements for products A,B

	A	B	Av.ity	M.V
	x_A	x_B		
RM1	2	1	15	π_1
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M.V. IN PLANNING:

Should new product C be manufactured if it requires, say 2 units RM1, 1 unit RM2, and 1 unit RM3?

Marginal values

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M.V. IN PLANNING:

Should new product C be manufactured if it requires, say 2 units RM1, 1 unit RM2, and 1 unit RM3?

Yes, if the sale profit of C is $> 2\pi_1 + \pi_2 + \pi_3$.