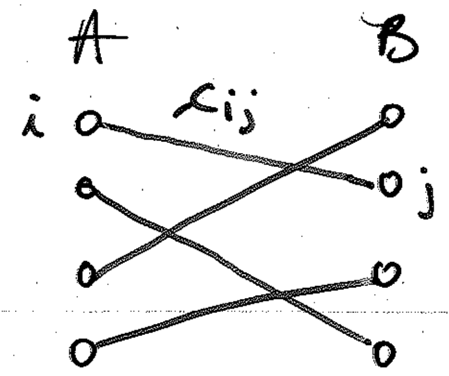
Assignment



min assignment cost.

$$(P)\begin{cases} \min \sum_{\substack{i\in A\\ j\in B}} c_{ij}\, x_{ij} \\ \sum_{j\in B} x_{ij} = 1 \quad , i\in A \\ \sum_{i\in A} x_{ij} = 1 \quad , j\in B \\ x_{ij} \geq 0 \end{cases}$$

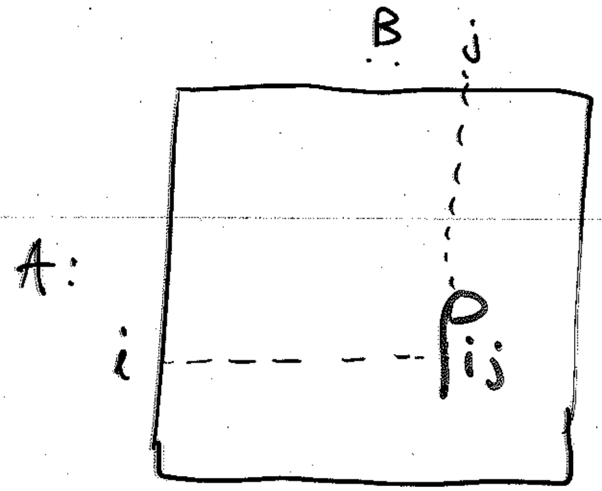
$$(D)\begin{cases} \max \sum_{i\in A} \alpha_i + \sum_{j\in B} \beta_j \\ \alpha_i + \beta_j \leq c_{ij} \quad , \begin{matrix} i\in A \\ j\in B \end{matrix} \end{cases}$$

Reduced cost (slack of dual constraint)

$$\rho_{ij} = c_{ij} - \alpha_i - \beta_j$$

C.S. : $\quad x_{ij} \cdot \rho_{ij} = 0.$

Primal-dual algorithm



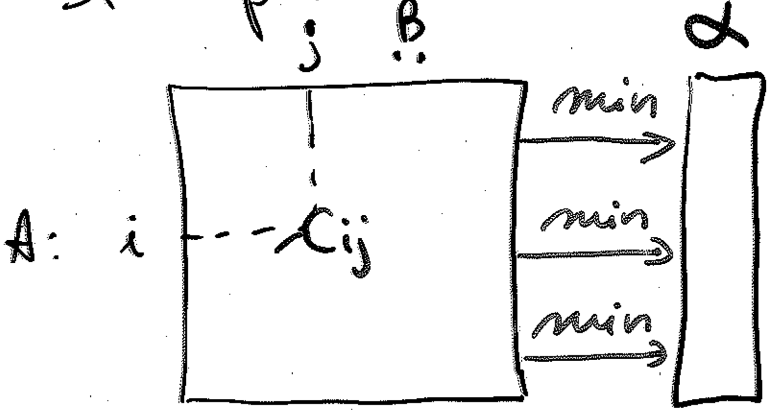
- dual feasibility
  $\rho_{ij} \geq 0$
- C.S.:
  assign i to j
  ($x_{ij} = 1$) only if
  $\rho_{ij} = 0$

$\underbrace{}_{\text{maintained throughout alg.}}$

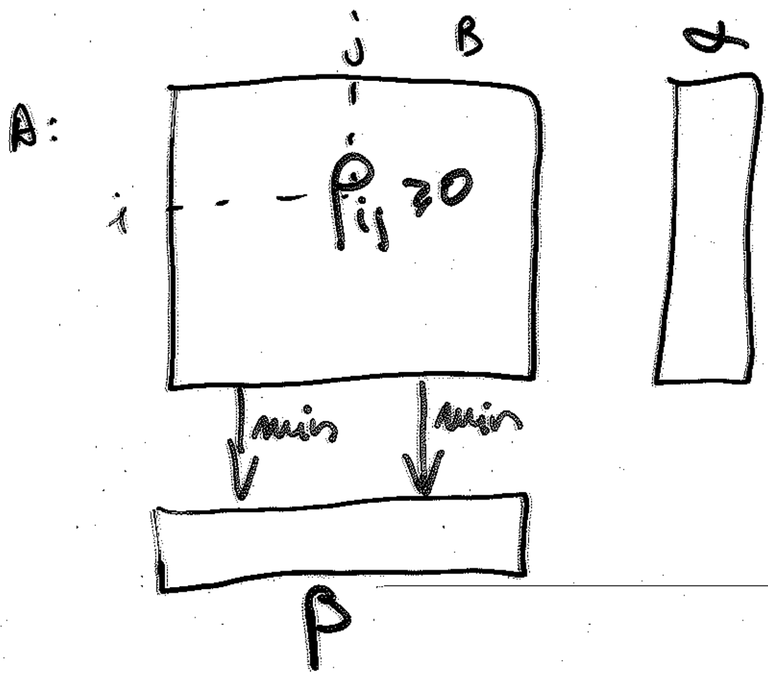
- primal feasibility:
  attempted through combinatorial
  methods (select 0 cells for assign.)

Primal-dual (c'ed).

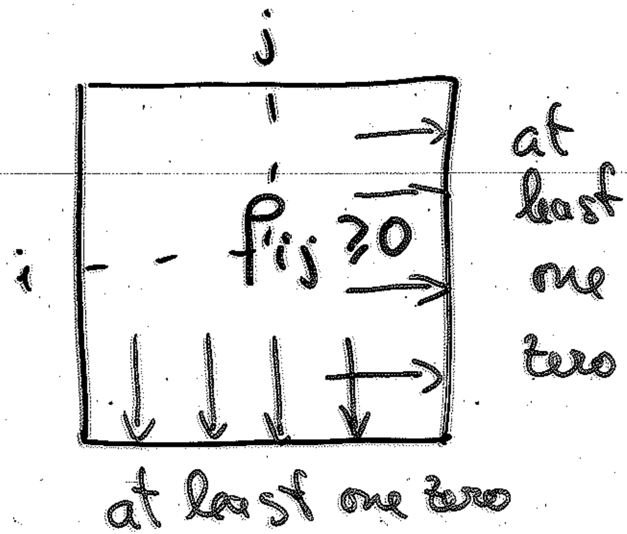
Startup:



$\beta = 0.$



Result:

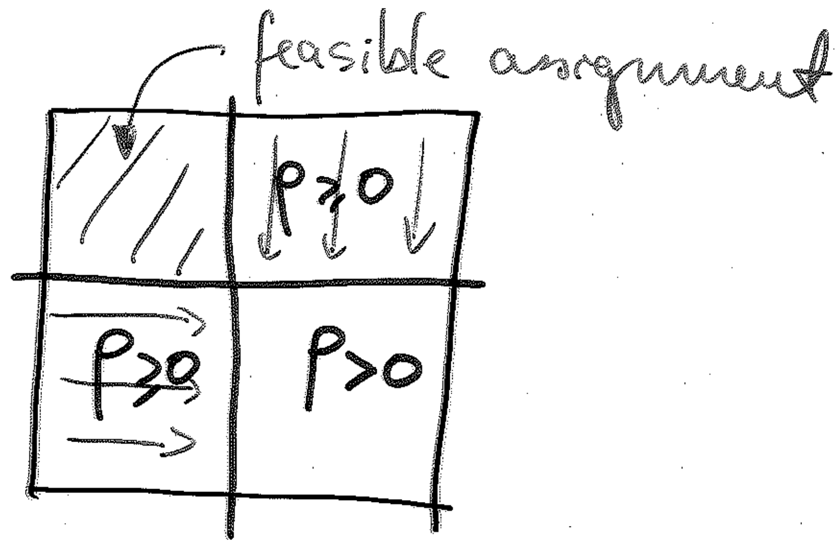


Primal-dual (c'ed)

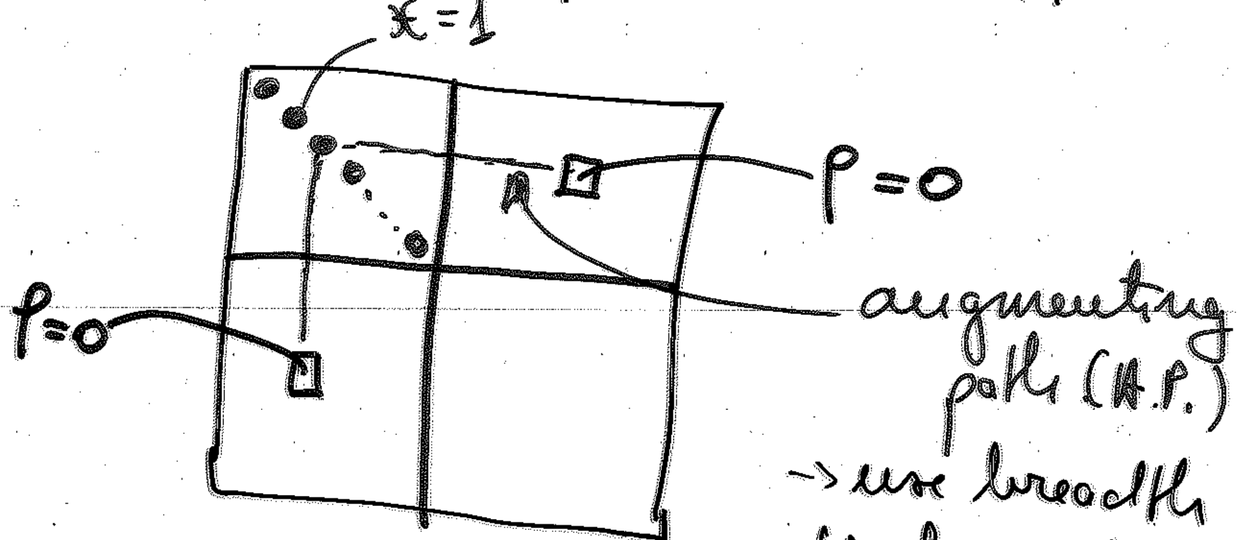
iteration: dual variables fixed (ρ_{ij}), increase primal feasibility.

a) Greedy algorithm
- select cell $\rho_{ij} = 0$
- assign $x_{ij} = 1$ if no conflicts

b)



- extending primal solution:

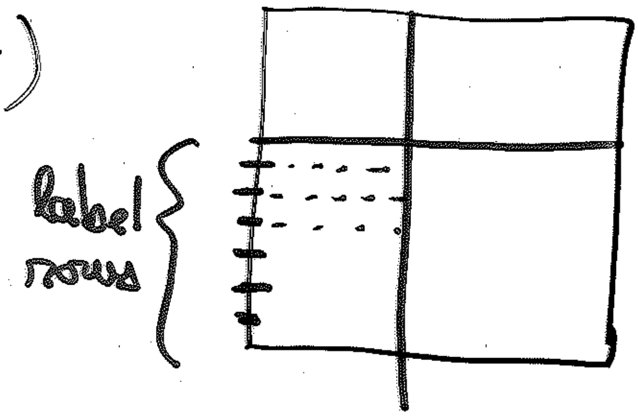


→ use breadth first search to find A.P

Primal-dual (c'ed).

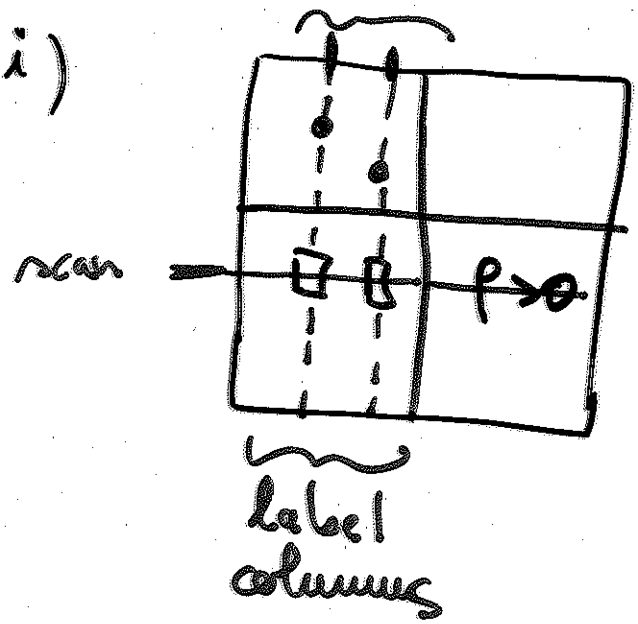
c) Breadth first search.

i)



- add labelled rows to queue.

ii)



- row scan

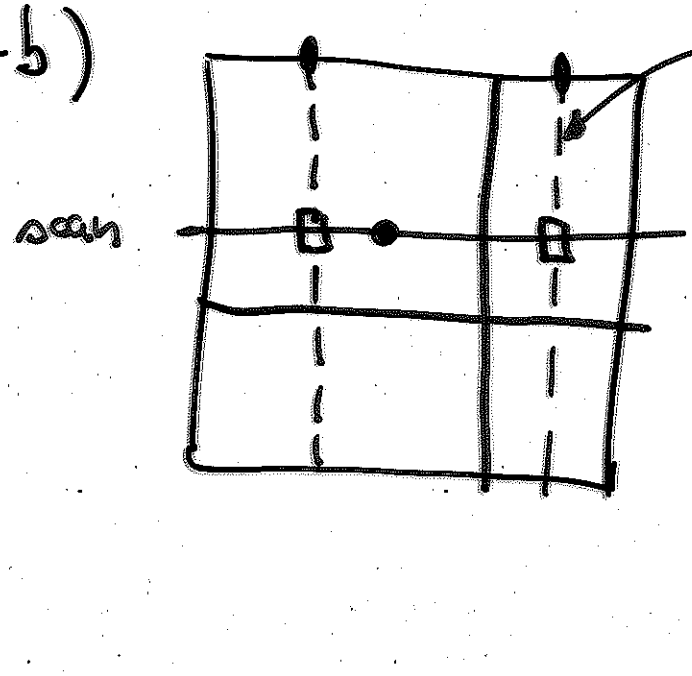
$\square : \rho = 0$

Label cols with $\rho = 0$ (if unlabeled)

Add to queue

ii-b)

no allocation



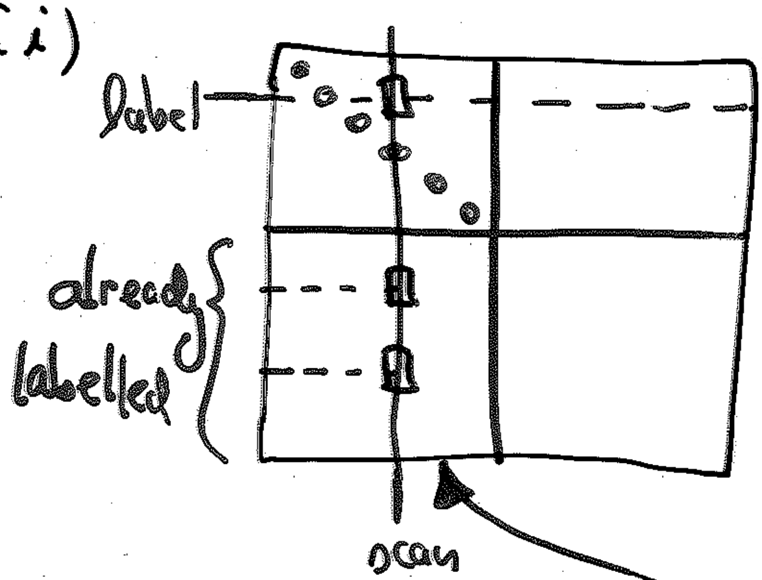
- row scan:

If a column is labeled & has no allocation,

AP found!

c) Breadth first search (c'ed)

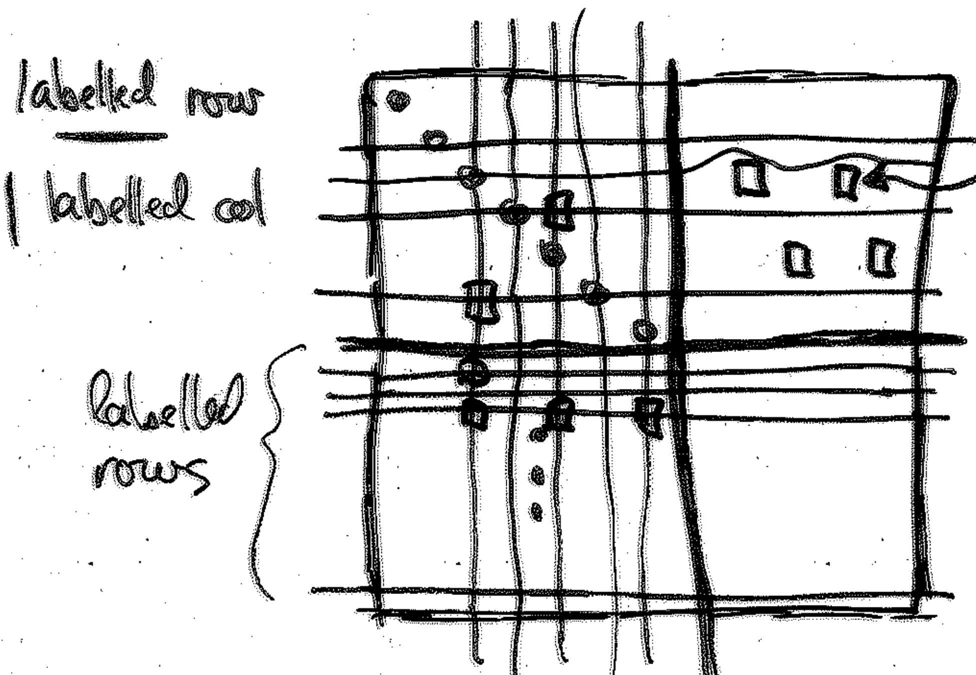
iii)



- column scan: Label unlabelled rows; Add them to queue.

Labelled rows & cols = rows & cols reachable from region • by following row-col paths through ($\rho = 0$) cells.

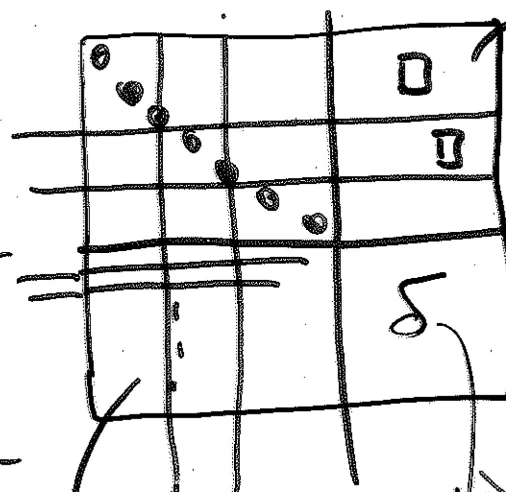
No ~~bno~~ A.P.



All $\rho = 0$ cells are NOT on labelled rows (otherwise A.P.!)

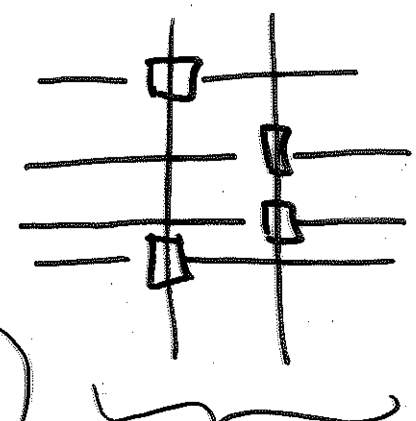
Primal-dual (c'ed)

d) Change dual:

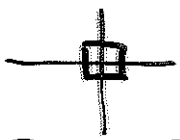


All rows labelled

No $\rho=0$ cells on labelled rows



All $\rho=0$ cells on labelled rows sit on lab. cols.

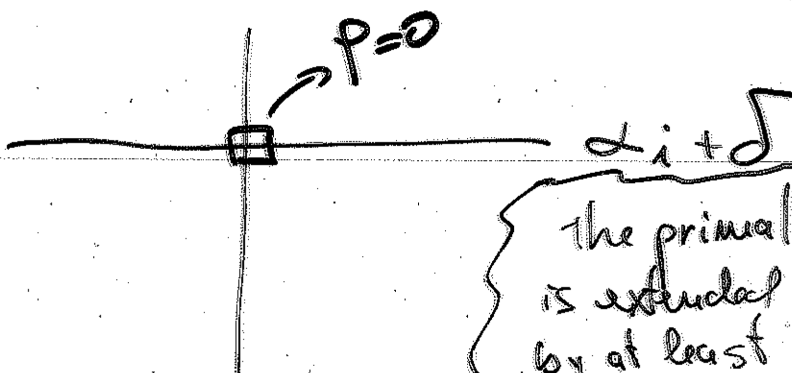


~~$\delta = \min \rho_{ij}$~~

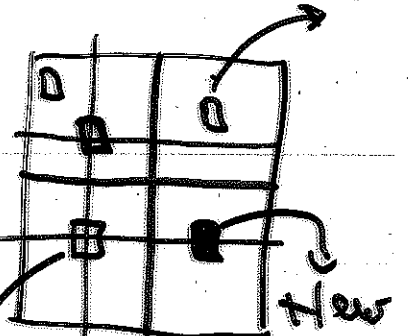
incorrect

All $\rho=0$ cells of labelled cols sit on labelled rows bc, lab. cols = <u>scanned</u>!

New duals:

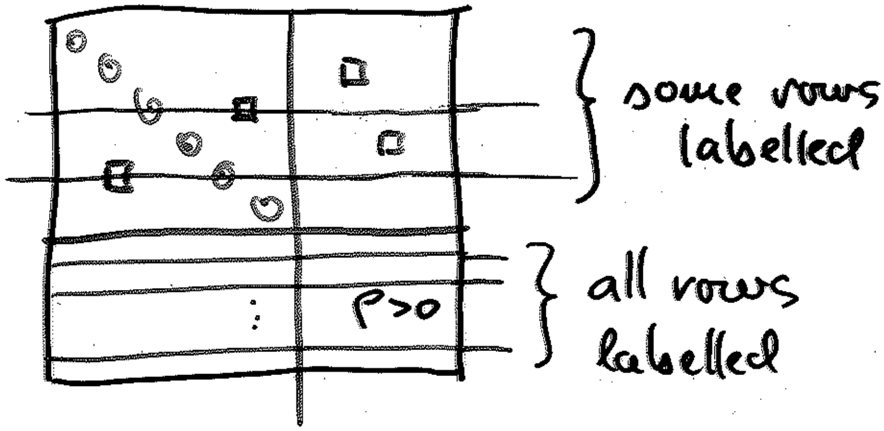


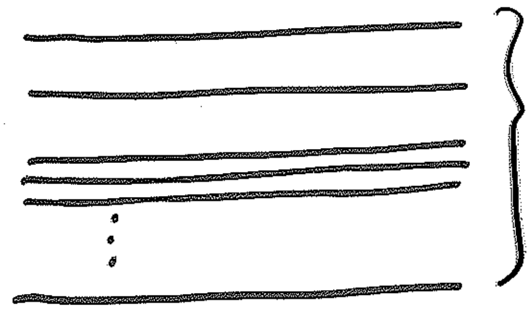
the primal is extended by at least one assignment



Primal-dual (c'ed)

d) Dual change routine:





$$\delta = \min\{\rho_{ij} \mid (i,j) \in -, \ \rho_{ij} > 0\}.$$

The update for dual variables is as on p. 7, except we take the min ρ_{ij} among all labelled rows (no cell on labelled rows becomes infeasible for dual pb.)

<u>Result</u>
- # of $(\rho_{ij} = 0)$ cells increases by 1 at least (primal assignment does not necessarily increase by 1 pair!)
- $O(n^2)$ iterations