Midterm solutions

① (Min weight spanning trees)

a) The constraint models the connectivity of the vertices.

b) $\max \sum_{S \subseteq V} y_S$

$$\sum_{\substack{S \\ e \not\subseteq S \\ e \cap S \neq \emptyset}} y_S \leq l(e) \qquad \forall e \in E$$

$$y_S \geq 0$$

c) The graph is K_4 with 6 edges & 4 vertices.

$\Rightarrow$ 6 variables

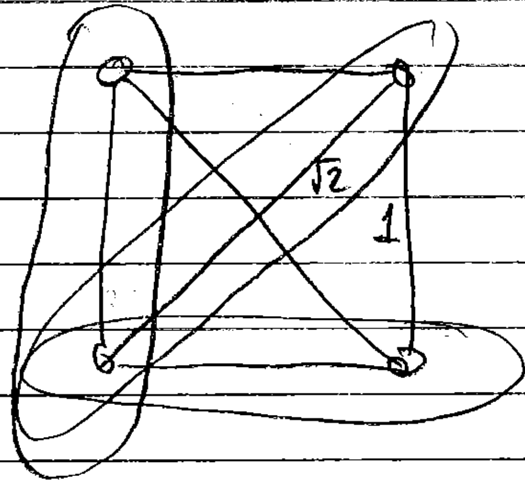
There is a constraint for every subset $S \subseteq V$ of vertices except for $S = V$ and $S = \emptyset$

$\Rightarrow 2^4 - 2 = 16 - 2 = 14$ constraints.

You may have noticed that the constraints generated by some set $S \subseteq V$ is the same as the one generated by $V - S$, so answering 7 is also correct.

Midterm sol (p 2)

(2) It helps to remove the redundant
a) constraints from the LP & think in terms of only the 7 constraints: the 4 sets of cardinality 1 and 3 sets of cardinality 2. The sets of cardinality 2 are circled.



The OPT solution has cost 2 ($\frac{1}{2}$ on each of the nodes of the square).

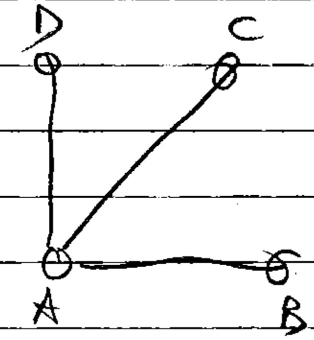
The optimal dual solution assigns $\frac{1}{2}$ to each set of cardinality 1 with cost also 2. To verify feasibility of both primal and dual solutions observe that for any of the three sets S of cardinality 2 circled, $\delta(S) \geq 1$.
The dual is obviously feasible (every edge has 2 endpoints and the sum of dual variables of the sets not containing the edge but touching its endpoint is $1 \leq \ell(e)$).

Midterm - sol (p3)

(2.b) The outcome of the primal-dual depends on how the dual variables are modified and how the complementary slackness conditions are used.

Important is to maintain dual feasibility and use Compl Slackness conditions somehow.

Ex



setting $y_{\{A\}} = 1$ allows one to choose the primal in the figure (Compl Slackness are satisfied), and the procedure stops.

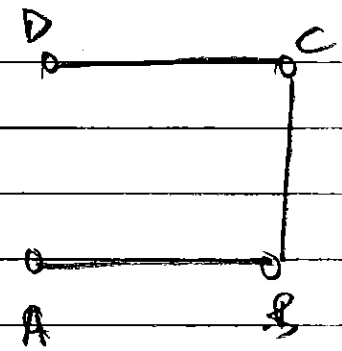
The optimal solution is obtained if y_A, y_B, y_C, y_D are increased in parallel.

Midterm sol (p 4)

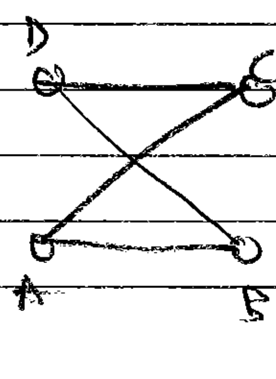
(3) If $\alpha < \frac{3}{2}$ then one can use the α-approx to answer the Hamiltonian path problem which is NP-c.

(4) Consider the optimal Steiner tree T_S. Traverse T_S and shortcut any edges that are not between two terminals like for TSP. You obtain a spanning tree which is no worse than 2× the cost of the Steiner tree (2× because of the traversal). Then the min. weight spanning tree over the terminals is not worse than 2× the optimal too.

(5)



Consider the unit square. A simple example depends on the order of vertices on the MST traversal. If the MST shown is traversed from vertex B to A then back at B, towards C etc, we get:



Cost $2 + 2\sqrt{2} = 2(1+\sqrt{2}) > 4$

the ratio is $\frac{1+\sqrt{2}}{2}$ for this example.

Midterm sol (p5)

(6) Given G, we show how to construct a set cover problem solving the dominating set problem.

Universe = set V of vertices of G
Collection of subsets $\{ N(v) : v \in V \}$ where

$N(v)$ is the set of neighbours of v, including v itself.