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Approximating Shortest Superstring Problem Using de Bruijn Graphs

Advanced Algorithms Course Presentation

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- This presentation is a review of the following paper:

  Title: Approximating Shortest Superstring Problem Using de Bruijn Graphs
  Authors: Alexander Golovnev, Alexander S. Kulikov, and Ivan Mihajlin

- This paper presents some results on approximating the shortest superstring problem.

Problem definition

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common superstring problem (SCS) :
given: n strings $s_1, \ldots, s_n$
goal: find a shortest string containing each s_i as a substring.

Definition

r-superstring problem (r-SCS) : SCS problem for the special case when all input strings have length exactly r.

Complexity

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- SCS over the binary alphabet and 3-SCS are NP-hard.
- 2-SCS can be solved in linear time.

Previous results for approximating SCS

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3	Blum, Jiang, Li, Tromp and Yannakakis [4]	1991
$2\frac{8}{9}$	Teng, Yao [23]	1993
$2\frac{5}{6}$	Czumaj, Gasieniec, Piotrow, Rytter [8]	1994
$2\frac{50}{63}$	Kosaraju, Park, Stein [15]	1994
$2\frac{3}{4}$	Armen, Stein [1]	1994
$2\frac{50}{69}$	Armen, Stein [2]	1995
$2\frac{2}{3}$	Armen, Stein [3]	1996
$2\frac{25}{42}$	Breslauer, Jiang, Jiang [5]	1997
$2\frac{1}{2}$	Sweedyk [21]	1999
$2\frac{1}{2}$	Kaplan, Lewenstein, Shafrir, Sviridenko [12]	2005
$2\frac{1}{2}$	Paluch, Elbassioni, van Zuylen [18]	2012
$2\frac{11}{23}$	Mucha [16]	2013

Result presented in this paper

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- In this paper a $\frac{r^2+r-4}{4r-6}$-approximation to r-SCS is presented.
- This is better than the best known approximation ratio ($2\frac{11}{23}$) for $r = 3, \ldots, 7$.

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Definition

overlap(s, t): the longest suffix of s that is also a prefix of t.

Definition

prefix(s, t): first $|s| - |overlap(s, t)|$ symbols of s.

Definition

suffix(s, t): last $|t| - |overlap(s, t)|$ symbols of t.

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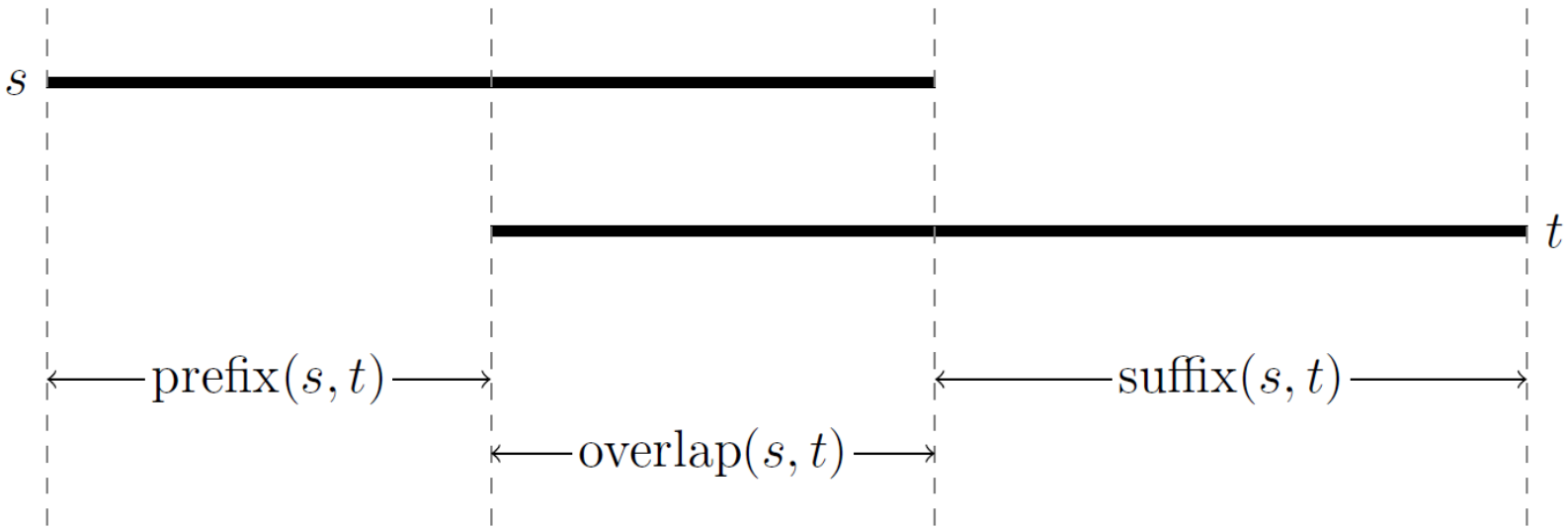

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Definition

prefix(s): string resulting from s by removing the last symbol.

Definition

suffix(s): string resulting from s by removing the first symbol.

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let $S = \{s_1, \ldots, s_n\}$ be a set of strings over an alphabet Σ and s be a superstring of S. The **compression** of s (w.r.t. S) is : $|s_1| + |s_2| + \cdots + |s_n| - |s|$.

- Clearly minimizing the length of a superstring corresponds to maximizing the compression.

SCS as a typical permutation problem

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- If we know the order of the input strings in a shortest superstring then we can recover this superstring by overlapping the strings in this given order.
- For this reason, it will be convenient for us to identify a superstring with the order of input strings in it.

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Definition

The **overlap graph** OG(S) of the set of strings $S = \{s_1, \ldots, s_n\}$ is a complete weighted directed graph on a set of vertices $V = \{1, \ldots, n\}$. The weight of an edge from i to j equals $|overlap(si, sj)|$.

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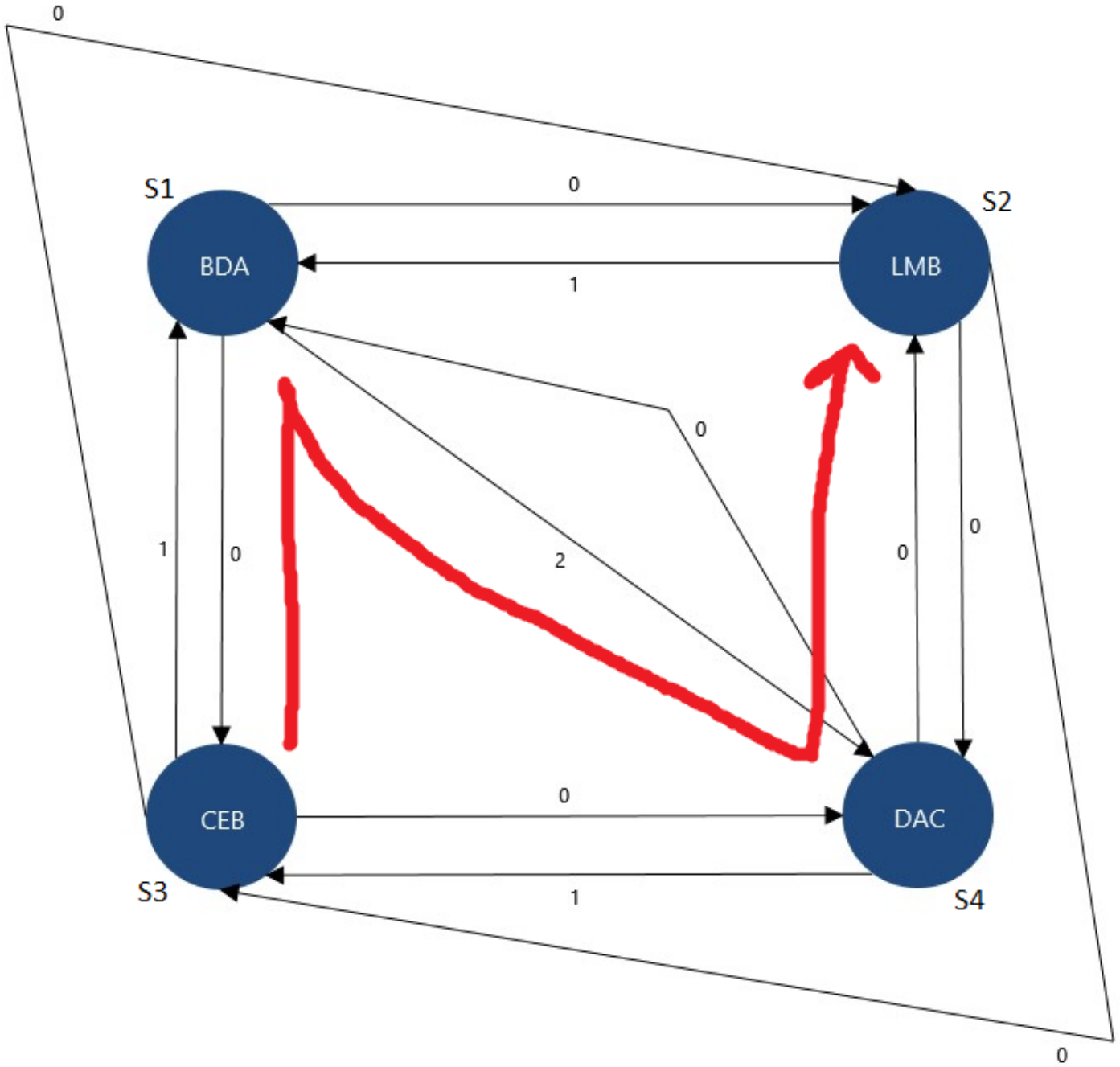

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asymmetric maximum traveling salesman path problem (MAX-ATSP) is to find a longest path visiting each vertex of the graph exactly once (such a path is called Hamiltonian).

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- Solving SCS corresponds to solving the asymmetric maximum traveling salesman path (MAX-ATSP) problem in overlap graph OG(S).
- The length of any Hamiltonian path in this graph equals the compression of the corresponding superstring.
- The best known approximation ratio for MAX-ATSP is $\frac{2}{3}$.
- This immediately gives a $\frac{2}{3}$-approximation for the compression.
- An α-approximation for MAX-ATSP (compression) implies a $3.5 - 1.5\alpha$ approximation for SCS.
- This means a 2.5-approximation for SCS.

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Definition

In **De Bruijn Graph** each input string $s_i \in S$ is represented as a directed (unweighted) edge from prefix(s_i) to suffix(s_i).

2-SCS using De Bruijn Graph and eulerian path

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- 2-SCS can be solved using De Bruijn Graph.
- An eulerian path spells a shortest superstring.
- To make the graph have an eulerian path, for each weakly connected component we add edges between imbalanced vertices (i.e., vertices with non-zero difference of in-degree and out-degree) so that the resulting component contains an Eulerian path. Finally, we add edges between components so that the graph contains an Eulerian path.
- r-SCS cannot be solved with the same technique because in case of 2-SCS, strings from different weakly connected components do not share letters (and hence have empty overlap) so the components can be traversed in any order.

2-SCS using De Bruijn Graph and eulerian path

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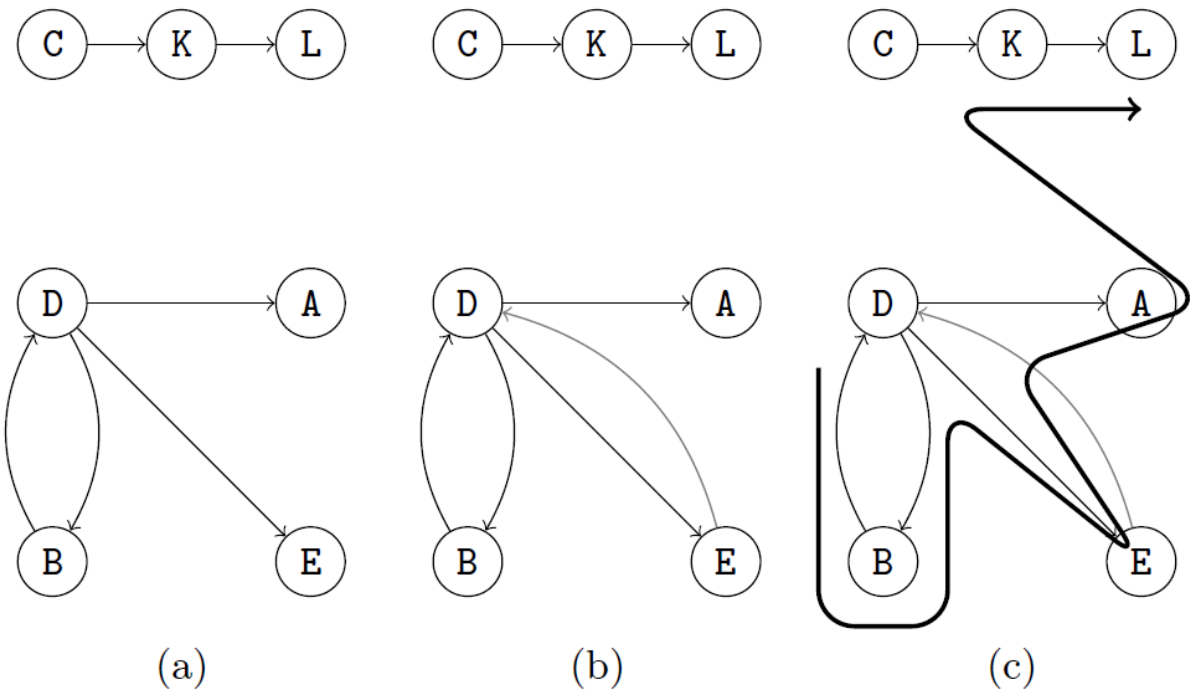

Fig. 2. 2-SCS can be solved in polynomial time. (a) de Bruijn graph of a set of strings {KL, DB, DE, CK, BD, DA}. (b) After adding an edge ED each weakly connected component contains an Eulerian path. (c) The string DBDEDACKL spelled by a path going through all the edges is a shortest superstring.

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Algorithm 3.1. $(r^2 + r - 4)/(4r - 6)$-approximation algorithm r-SCS

Input: $\mathcal{S} = \{s_1, \ldots, s_n\} \subseteq \Sigma^r$.

Output: A superstring of S that is at most $(r^2 + r - 4)/(4r - 6)$ times longer than a shortest superstring.

// first, find a long traveling salesman path in the overlap graph

1: let π be a 2/3-approximate maximum traveling salesman path in $\mathrm{OG}(\mathcal{S})$

// then, find a short rural postman path in the de Bruijn graph

2: let $\mathcal{S}' = \{s'_1, \ldots, s'_n\} \subseteq \Sigma_1^2$ be a set of 2-strings over the alphabet $\Sigma_1 = \Sigma^{r-1}$; s'_i is the 2-string consisting of prefix of s_i of length $r - 1$ and suffix of s_i of length $r - 1$

3: let π_1 be a shortest superstring for the set of 2-strings $\mathcal{S}'$

4: **return** the better one among π and π_1

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Theorem

Algorithm 3.1 finds an $\alpha(r)$-approximation for r-SCS where $\alpha(r) = \frac{r^2+r-4}{4r-6}$

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- Let H be a shortest Hamiltonian path in OG(S). Then clearly
  $OPT(S) = rn - w(H)$,
  where w(H) is the weight of H.
- A $\frac{2}{3}$-approximate maximum traveling salesman path has weight at least $\frac{2w(H)}{3}$.
- Thus, the permutation Π gives a superstring of length at most $rn - \frac{2w(H)}{3}$
- The corresponding approximation ratio is:

$$\frac{rn - \frac{2w(H)}{3}}{rn - w(H)}. \qquad (1)$$

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- Let u denote the number of edges of weight at most (r - 2) in H.
- Then the number of edges of weight exactly (r - 1) in H is (n - 1 - u).
- Then we have

$$w(H) \leq (r-1)(n-1-u) + (r-2)u \qquad \text{(a)}$$
$$w(H) \leq (r-1)(n-1-u) + (r-1)u - u \qquad \text{(b)}$$
$$w(H) \leq (r-1)(n-1) - u \qquad \text{(c)}$$

and hence

$$u \leq (r-1)(n-1) - w(H). \qquad (2)$$

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- Note that

$$overlap(s'_i, s'_j) = \begin{cases} 1 & \text{if } overlap(s_i, s_j) = r - 1 \\ 0 & \text{otherwise.} \end{cases}$$

- Since S' is a 2-SCS instance, a shortest superstring for S' has the maximal possible number of overlaps of size 1.
- This number is in turn equal to the maximal possible number of overlaps of size r-1 for S.
- Since the number of overlaps of size r-1 in H is (n-1-u), the length of a shortest superstring for S' is at most $2n - (n - 1 - u) = n + u + 1$.

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- Hence Π_1 gives a superstring of S of length at most
  $rn - (r-1)(n-1-u)$ (a)
- Because of (2), this is at most
  $rn - (r-1)(n-1-(r-1)(n-1)+w(H))$ (b)
  which is less than
  $(r^2-2r+2)n-(r-1)w(H).$ (c)
- The corresponding approximation ratio is

$$\frac{(r^2-2r+2)n-(r-1)w(H)}{rn-w(H)} \quad (3)$$

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- From (1) and (3) and a simple observation that $0 \leq \frac{w(H)}{n} \leq (r-1)$ we conclude that the approximation ratio of the constructed algorithm is

$$\alpha(r) = \max_{0 \leq x \leq r-1} \{\min\{\frac{r-\frac{2x}{3}}{r-x}, \frac{(r^2-2r+2)-(r-1)x}{r-x}\}\}$$

- The maximum of their minimum is attained at x where they meet, namely

$$x = \frac{r^2-3r+2}{r-\frac{5}{3}}.$$

- Plugging in this x gives

$$\alpha(r) = \frac{r^2+r-4}{4r-6}.$$