

### Math 1410–Assignment 3

Due Friday (October 7, 2005) before the lecture in the class

1. Find  $2A - A^2$ , where

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}.$$

2. Find the values of  $x, y$  that make  $AB + A = 0$ , where

$$A = \begin{pmatrix} x & 1 \\ 1 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 1 & 2 \\ y & 1 \end{pmatrix}.$$

3. Find  $A^3 - 4A^2 + 4A$ , where  $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix}$ .

4. write the following system of equations in matrix form  $AX = B$  and identify the matrices  $A, X$  and  $B$ :

$$\begin{aligned} 3x^2 - 2y + 3z^3 + w &= 7 \\ 2x^2 - 3y - 4z^3 - w &= 3 \\ 11x^2 + y + 4z^3 - w &= 31 \\ x^2 + y - 2z^3 + w &= 11 \end{aligned}$$

5. If  $A, B$ , and  $C$  are any  $2 \times 2$  matrices, prove or disprove the following:

- (a)  $AB - BA = 0$  for all  $A$  and  $B$ .  
(b)  $A^3 = 0$  implies  $A = 0$ .  
(c)  $BC = 2AC$  implies  $B = 2A$  for all  $A, B$  and  $C$ .  
(d) If  $AB = BA$ , then  $(A - B)^2 = A^2 - 2AB + B^2$ .

6. Let

$$A = \begin{pmatrix} 1 & 1 & -1 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & -1 & 1 \\ 1 & 2 & -1 & 1 \end{pmatrix}.$$

Find elementary matrices  $E_1, E_2, \dots, E_n$ , such that  $E_n \cdots E_2 E_1 A$  is the reduced echelon form of  $A$ .

7. Let  $A = \begin{pmatrix} 1 & 0 \\ a & 1 \end{pmatrix}$ , where  $a$  is any number. Find a  $2 \times 2$  matrix  $B$  such that  $BA = I$  and find  $AB$ .