Math 1410–Assignment 4

Due Friday (Oct. 14, 2005) before the lecture in the class

1. For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix}$, find a matrix B such that BA is the the reduced echelon form of A.

2. Find the inverse matrix (if there is one) of each of the following matrices:

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & 1 \\ 3 & 1 & 4 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

3. Write the following system of equations in matrix form $AX = B$ and then use it to solve the system (note that the matrix A is the same as in problem 2):

$$\begin{array}{rcrcrcl} 2x & + & y^2 & + & z^3 & = & 2 \\ x & & & + & z^3 & = & 0 \\ 3x & + & y^2 & + & 4z^3 & = & 0 \end{array}$$

4. Find B^{-1} if

$$A = \begin{bmatrix} 1 & -2 & 1 \\ 0 & 1 & -3 \\ 2 & 1 & 0 \end{bmatrix}, \quad (AB)^{-1} = \begin{bmatrix} -2 & 1 & -6 \\ 3 & 1 & 10 \\ 6 & 2 & -4 \end{bmatrix}.$$

5. Let P be a matrix such that $PP^t = nI$, where n is a nonzero number. Show that

$$P^{-1} = \frac{1}{n}P^t.$$

   Use this to find P^{-1}, if

$$P = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}.$$

6. (Bonus problem) A is a 2×2 matrix. Show that if $AB = BA$ for all 2×2 matrices B, then $A = aI$ for some number a.