Math 1410–Assignment 5

Due Friday (October 21, 2005) before the lecture in the class

1. Evaluate each of the following determinants:

$$\begin{vmatrix} 1 & 3 & 0 \\ 5 & -4 & 1 \\ -1 & 2 & 1 \end{vmatrix}, \quad \begin{vmatrix} -1 & 1 & 6 & 1 \\ 1 & 5 & 3 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & -1 & 3 & 1 \end{vmatrix}.$$

2. Evaluate the following determinant using the cofactor expansion along the first row. Also, compute the determinant using the cofactor expansion down the second column.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & -6 \\ -7 & 8 & 1 \end{vmatrix}.$$

3. Find the inverse of each of the following matrices using the cofactor method:

$$\begin{bmatrix} 1 & -1 & 5 \\ 1 & 1 & 1 \\ 3 & -4 & 2 \end{bmatrix}, \quad \begin{bmatrix} 1 & -2 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 0 & 0 & 2 & -7 \\ 0 & 0 & 3 & 0 \end{bmatrix}.$$

4. Find all values of a, b, c and d for which the following matrix is singular (a square matrix A is called singular if $|A| = 0$):

$$\begin{bmatrix} 2a & -1 & 1 & 1 \\ b & 2 & 1 & 1 \\ 0 & 0 & c & d \\ 0 & 0 & 1 & -4 \end{bmatrix}.$$

5. Prove or disprove each of the following statements:

(a) $|A+B| = |A| + |B|$, for all (square) matrices A and B.

(b) $|2A| = 8|A|$, for all 3 by 3 matrices A.

(c) $\left|(AB)^{-1}\right| = \frac{1}{|A||B|}$, for all nonsingular (square) matrices A and B.

6. Use the Cramer's rule to solve the following system of linear equations for any values of $a \neq 0$ and $b \neq 2$:

$$\begin{array}{rcrcrcr} x & + & ay & + & bz & = & 10 \\ x & + & ay & + & 2z & = & 2 \\ 2x & + & ay & + & 3z & = & 5 \end{array}$$