Math 1410–Assignment 6

Due Friday (Nov. 4, 2005) before the lecture in the class

1. Given the vectors $\underline{a} = \{(1,1,1), \underline{b} = (-2,-1,2)\}$, and $\underline{\text{v}} = (3,4,7)$. Express the vector $\underline{\text{v}}$ as a linear combination of $\underline{a}$ and $\underline{b}$

2. Show that the $span\{(-1,0,1),(-2,0,-2)\}$ consists of all three dimensional vectors with the second component 0.

3. Given $S = \{(1,0,1,1),(0,1,1,1),(1,1,0,1)\}$,

   (a) Determine whether the vector $\underline{\text{v}} = (-2,-3,3,-1)$ is in the span of S, and

   (b) if the vector is in the sapn, write it as a linear combination of elements of S.

4. Which of the following subsets are subspaces of $\mathbb{R}^3$? You need to justify your claim.

   (a) $\{(x,0,z) : \ x = 3z\}$

   (b) $\{(x,y,z) : \ x+3y-2z = 5\}$

   (c) $\{(x,y,z) : \ 3x+3y-z = 0\}$

   (d) $\{(x,y,z) : \ xy+z = 0\}$

   (e) $\{(x,y,z) : \ x^2 = 0\}$

   (f) $\{(x,y,z) : \ x+3y-z = 0 \text{ and } 2x-3y+z = 0\}$

5. Consider the system of the linear equations $AX = 0$, where A is any $n \times m$ matrix and X a column vector of dimension m. Show that

$$S = \{X : \ AX = 0\},$$

   the solution set of the system, is a subspace of $\mathbb{R}^m$.

6. (Bonus) A plane in $\mathbb{R}^3$ is the set of all vectors (x,y,z) satisfying an equation like $ax+by+cz = d$. Show that a plane is a subsapce if and only if it contains the origin.