Math 1410–Assignment 8

Due Friday Nov. 25, 2005 before the lecture in the class

1. Let $A = \begin{bmatrix} 1 & -1 & 1 & 1 & -1 \\ 0 & 1 & 1 & 1 & 1 \\ -1 & 2 & 0 & 0 & 2 \end{bmatrix}$. Find the dimension of:

   (a) the row space of A,

   (b) the solution set of the equation $A\underline{x} = 0$ (note that $\underline{x}$ is a column vector).

2. Let $\underline{v} = (-2,5,2,4)$ and $\underline{u} = (1,1,0,-1)$.

   (a) Find the projection of $\underline{v}$ on $\underline{u} = (1,1,0,-1)$ and call it $\underline{w}$.

   (b) Find the vector $\underline{x} = \underline{v} - \underline{w}$.

   (c) Find $\underline{u} \circ \underline{x}$.

3. (a) Show that the two vectors $\underline{u} = (1,1,1)$ and $\underline{v} = (1,-2,1)$ are orthogonal.

   (b) Let $\underline{w} = (1,0,1)$. Find $\text{proj}_{\underline{u}}\underline{w}$ and $\text{proj}_{\underline{v}}\underline{w}$.

   (c) Verify that $\underline{w} = \text{proj}_{\underline{u}}\underline{w} + \text{proj}_{\underline{v}}\underline{w}$.

4. (a) Verify that the set of vectors

   $$S = \{(1,1,1,1),\ (1,-1,1,-1),\ (1,-1,-1,1),\ (1,1,-1,-1)\}$$

   forms an orthogonal basis for $\mathbb{R}^4$.

   (b) Use part (a) to express the vector $(1,2,3,4)$ as a linear combination of the vectors in S.

5. Let $\underline{a} = (-3,2,1)$, $\underline{b} = (1,1,1)$, and $\underline{c} = (9,-4,7)$ be vectors in $\mathbb{R}^3$.

   (a) Show that vector $\underline{a}$ is orthogonal to the vector $\underline{b}$.

   (b) Let $\underline{u} = \text{proj}_{\underline{a}}\underline{c}$ and $\underline{v} = \text{proj}_{\underline{b}}\underline{c}$. Find the vector $\underline{w} = \underline{c} - \underline{u} - \underline{v}$.

   (c) Show that vector $\underline{w}$ is orthogonal to both vectors $\underline{a}$ and $\underline{b}$.