Math 1410–Assignment 9

Due Friday Dec. 2, 2005 before the lecture in the class

1. (a) Verify that the three vectors $\underline{u} = \left(\frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}}\right)$, $\underline{v} = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}\right)$, $\underline{w} = \left(\frac{-1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ form an orthonormal basis for $\mathbb{R}^3$.

   (b) Express the vectors $(1, -3, 4)$ and $(2, 1, 2)$ as linear combinations of the above basis.

2. (a) Use the Gram-Schmidt process to orthonormalize the vectors

$$(1, 1, 1, 1), (1, 1, 1, -1), (1, 2, 2, 0).$$

   (b) Use part (a) to see if the vector $(1, 2, 2, -2)$ is in $\operatorname{span}\{(1, 1, 1, 1), (1, 1, 1, -1), (1, 2, 2, 0)\}$.

   (c) Repeat part (b) for the vector $(1, 2, 4, -2)$.

3. (a) Find the eigenvalues of the matrix

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 3 \\ 0 & 3 & 1 \end{bmatrix}.$$

   (b) Find a basis for each of the eigenspaces of the matrix A.

   (c) Orthonormalize the vectors found in (b) by applying the Gram-Schmidt process, if necessary.

   (d) Use the vectors found in (c) to form an orthogonal matrix P diagonalizing A.

   (e) Find the entry in the first row and first column of A^7.

4. Repeat Problem 3 for the matrix

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}.$$