1. (a) (5) Use the Gram-Schmidt process to orthonormalize the vectors (search for a better choice of first and second vector) $(1,1,1,1),\ (1,1,-1,1),\ (1,-1,1,1)$.

(b) Let $S = span\{(1,1,1,1),\ (1,1,-1,1),\ (1,-1,1,1)\}$. (7) Using part (a), determine if the vector $(-2,4,0,-2)$ is in S.

(c) (3) Is it true that $S = \mathbb{R}^4$? Explain

2. (a) (6) Verify that the three vectors $\underline{u} = (\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}, 0)$, $\underline{v} = (-\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}})$, $\underline{w} = (\frac{1}{\sqrt{6}}, -\frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}})$ form an orthonormal basis for $\mathbb{R}^3$.

(b) (4) Is it possible to find a vector which is not in the span of the vectors $\underline{u}$, $\underline{v}$ and $\underline{w}$?

3. Let

$$A = \begin{bmatrix} 5 & 1 & 1 \\ 1 & 5 & 1 \\ 1 & 1 & 5 \end{bmatrix}.$$

(a) (4) show that (1,1,1) is an eigenvector of A and find the corresponding eigenvalue.

(b) (4) show that (1,-1,0) is an eigenvector of A and find the corresponding eigenvalue.

(c) (6) Find all the eigenvalues of A.

(d) (10) Find a basis for each of the eigenspaces of the matrix A.

(e) (6) Find an orthogonal matrix P diagonalizing A.

(f) (6) Find The entry in the second row and second column of A^6.

4. (5,5,5) Which of the following subsets is a subspace of $\mathbb{R}^3$? You need to justify your claim.

(a) $\{(x, y, z) : \; x + 3y - z = 5\}$

(b) $\{(x, y, z) : \; x + 3y - z = 0\}$

(c) $\{(x, y, z) : \; xyz = 0\}$

5. Given $S = \{(-1, 0, 1), (-2, 0, 1)\}$.

(a) (4) Describe the vectors in the span of S.

(b) (7) Determine whether the vector $\underline{v} = (1, 0, 4)$ is in the span of S and if the vector is in the span, write it as a linear combination of elements of S

6. Let $A = \begin{bmatrix} 1 & -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & 1 & 1 \\ -1 & 1 & 3 & 3 & 1 \end{bmatrix}$.

(a) (5) Find the dimension of the row space of A.

(b) (8) Find a basis and the dimension of the solution set of the equation $A\underline{\mathrm{x}} = 0$.