

## Math 1410–Midterm Practice Sheet

Solutions will be posted on the evening of Thursday, October 27

1. Solve each system using Gaussian or Gauss Jordan elimination.

$$(a) \begin{cases} x^2 + y^3 - z^4 = 0 \\ x^2 + 2y^3 - 2z^4 = -9 \\ -x^2 - y^3 - z^4 = -2 \end{cases}$$

$$(b) \begin{cases} x - 2y = b \\ -2x + 4y = -6 \end{cases}$$

2. Solve the following matrix equation for  $A$ :

$$2A^{-1} - \begin{bmatrix} 1 & 0 \\ 3 & 3 \end{bmatrix}^t \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^t = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}^{-1}$$

3. Find elementary matrices  $E_1, E_2, \dots, E_n$  such that  $E_n E_{n-1} \dots E_2 E_1 A$  is the reduced echelon form of  $A$ . Also, find the products  $E_n E_{n-1} \dots E_2 E_1 A$  and  $E_n E_{n-1} \dots E_2 E_1$ .

$$A = \begin{bmatrix} 0 & -2 & 6 & -8 \\ -1 & 1 & -1 & 3 \end{bmatrix}$$

4. If  $A = \begin{bmatrix} 0 & 0 & -2 \\ -2 & 0 & 0 \\ 0 & -2 & 0 \end{bmatrix}$ , show that  $A^3 + 8I = 0$ . Use this to find  $A^{-1}$ .

5. Let  $A = \begin{bmatrix} 1 & 5 & 2 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -2 \\ 0 & 3 & 1 & 4 \end{bmatrix}$ .

- (a) Find the cofactors  $A_{13}$  and  $A_{43}$ .  
(b) Use the results above to find  $|A|$ .  
(c) What is the  $(3, 1)$ -entry of  $\text{adj}(A)$ ?  
(d) What is the  $(3, 4)$ -entry of  $A^{-1}$ ?

6. Use each technique below to solve the system  $\begin{cases} x + y = 1 \\ 3x + 2y = -1 \end{cases}$

- (a) Form the augmented matrix and find its reduced echelon form.  
(b) Form the matrix equation  $AX = B$  and use  $A^{-1}$  to find  $X$ .  
(c) Use Cramer's rule.