Math 1410–Final Exam Practice Sheet

Solutions will be posted by the evening of Friday, December 9

1. Let $A = \begin{bmatrix} 0 & 1 & -5 & 4 \\ 2 & 1 & -9 & -10 \\ -2 & 0 & 4 & 6 \end{bmatrix}$. Find a basis and the dimension of

   (a) the row space of A, and

   (b) the solution set of $A\underline{x} = 0$.

2. Show that $\underline{u}_1 = \left(\frac{3}{13}, \frac{4}{13}, \frac{12}{13}\right)$, $\underline{u}_2 = \left(\frac{4}{5}, -\frac{3}{5}, 0\right)$, and $\underline{u}_3 = \left(\frac{36}{65}, \frac{48}{65}, -\frac{25}{65}\right)$ form an orthonormal basis for $\mathbb{R}^3$.

3. Let $\underline{v}_1 = (1, 1, 1, 1)$, $\underline{v}_2 = (3, 1, 9, -5)$, and $\underline{v}_3 = (13, -3, 11, -1)$.

   (a) Use Gram-Schmidt to orthonormalize $\underline{v}_1$, $\underline{v}_2$, and $\underline{v}_3$.

   (b) Determine whether $\underline{a} = (8, 8, 0, 0)$ is in the span of $\{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$.

   (c) Determine whether $\underline{b} = (30, -30, -40, 40)$ is in span$\{\underline{v}_1, \underline{v}_2, \underline{v}_3\}$.

4. Let $A = \begin{bmatrix} -1 & -3 & 0 \\ 3 & 5 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 12 & 0 \\ 12 & -1 & 0 \\ 0 & 0 & 15 \end{bmatrix}$.

   (a) Show that $(1, -1, 0)$ is an eigenvector of A and find the corresponding eigenvalue.

   (b) Find all the eigenvalues of A.

   (c) Find a basis for each eigenspace of A.

   (d) Is A diagonalizable?

   (e) Find all the eigenvalues of B.

   (f) Find a basis for each eigenspace of B.

   (g) Orthonormalize the vectors found in (f), using Gram-Schmidt if necessary.

   (h) Use the vectors found in (g) to create a matrix P that diagonalizes B.

   (i) Find the (1,1)-entry in B^5.

5. Determine whether each statement is true or false. Justify your answer.

   (a) $(br - cq,\ cp - ar,\ aq - bp)$ is orthogonal to both (a, b, c) and (p, q, r).

   (b) $\{(x, y) : x^2 = xy\}$ is a subspace of $\mathbb{R}^2$.

   (c) If 0 is an eigenvalue of an $n \times n$ matrix A, then A is invertible.