Math 1410–Solutions for Assignment 5

Submitted Friday, October 21

1. Evaluate each of the following determinants:

$$\begin{vmatrix} 1 & 3 & 0 \\ 5 & -4 & 1 \\ -1 & 2 & 1 \end{vmatrix}.$$

Solution:

$$\begin{aligned}
& \begin{vmatrix} 1 & 3 & 0 \\ 5 & -4 & 1 \\ -1 & 2 & 1 \end{vmatrix} \\
\begin{matrix} = \\ -5\mathsf{R1}+\mathsf{R2} \\ \mathsf{R1}+\mathsf{R3} \end{matrix} \quad & \begin{vmatrix} 1 & 3 & 0 \\ 0 & -19 & 1 \\ 0 & 5 & 1 \end{vmatrix} \\
= \quad & (1)(-1)^{1+1} \begin{vmatrix} -19 & 1 \\ 5 & 1 \end{vmatrix} \\
= \quad & (-1)^2 \begin{vmatrix} -19 & 1 \\ 5 & 1 \end{vmatrix} \\
= \quad & \begin{vmatrix} -19 & 1 \\ 5 & 1 \end{vmatrix} \\
= \quad & (-19)(1)-(1)(5) \\
= \quad & -19-5 \\
= \quad & -24.
\end{aligned}$$

$$\begin{vmatrix} -1 & 1 & 6 & 1 \\ 1 & 5 & 3 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & -1 & 3 & 1 \end{vmatrix}.$$

Solution:

$$\begin{vmatrix} -1 & 1 & 6 & 1 \\ 1 & 5 & 3 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & -1 & 3 & 1 \end{vmatrix}$$

$$\underset{\substack{R1+R2 \\ R1+R3 \\ R1+R4}}{=} \begin{vmatrix} -1 & 1 & 6 & 1 \\ 0 & 6 & 9 & 2 \\ 0 & 1 & 6 & 2 \\ 0 & 0 & 9 & 2 \end{vmatrix}$$

$$\underset{-R4+R2}{=} \begin{vmatrix} -1 & 1 & 6 & 1 \\ 0 & 6 & 0 & 0 \\ 0 & 1 & 6 & 2 \\ 0 & 0 & 9 & 2 \end{vmatrix}$$

$$= (-1)(-1)^{1+1} \begin{vmatrix} 6 & 0 & 0 \\ 1 & 6 & 2 \\ 0 & 9 & 2 \end{vmatrix}$$

$$= -\begin{vmatrix} 6 & 0 & 0 \\ 1 & 6 & 2 \\ 0 & 9 & 2 \end{vmatrix}$$

$$= -\left[(6)(-1)^{1+1} \begin{vmatrix} 6 & 2 \\ 9 & 2 \end{vmatrix}\right]$$

$$= -6 \begin{vmatrix} 6 & 2 \\ 9 & 2 \end{vmatrix}$$

$$= -6[(6)(2) - (2)(9)]$$

$$= -6[12 - 18]$$

$$= -6[-6]$$

$$= 36.$$

2. Evaluate the following determinant using the cofactor expansion along the first row. Also, compute the determinant using the cofactor expansion down the second column.

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & -6 \\ -7 & 8 & 1 \end{vmatrix}.$$

Solution:

Along the first row:

$$\begin{aligned}
&\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & -6 \\ -7 & 8 & 1 \end{vmatrix} \\
= \quad & +(1)\begin{vmatrix} 5 & -6 \\ 8 & 1 \end{vmatrix} - (2)\begin{vmatrix} 4 & -6 \\ -7 & 1 \end{vmatrix} + (3)\begin{vmatrix} 4 & 5 \\ -7 & 8 \end{vmatrix} \\
= \quad & [(5)(1) - (-6)(8)] - 2\,[(4)(1) - (-6)(-7)] + 3\,[(4)(8) - (5)(-7)] \\
= \quad & [53] - 2\,[-38] + 3\,[67] \\
= \quad & 53 + 76 + 201 \\
= \quad & 330.
\end{aligned}$$

Down the second column:

$$\begin{aligned}
&\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & -6 \\ -7 & 8 & 1 \end{vmatrix} \\
= \quad & -(2)\begin{vmatrix} 4 & -6 \\ -7 & 1 \end{vmatrix} + (5)\begin{vmatrix} 1 & 3 \\ -7 & 1 \end{vmatrix} - (8)\begin{vmatrix} 1 & 3 \\ 4 & -6 \end{vmatrix} \\
= \quad & -2\,[(4)(1) - (-6)(-7)] + 5\,[(1)(1) - (3)(-7)] - 8\,[(1)(-6) - (3)(4)] \\
= \quad & -2\,[-38] + 5\,[22] - 8\,[-18] \\
= \quad & 76 + 110 + 144 \\
= \quad & 330, \quad \text{as expected.}
\end{aligned}$$

3. Find the inverse of each of the following matrices using the cofactor method:

$$\begin{bmatrix} 1 & -1 & 5 \\ 1 & 1 & 1 \\ 3 & -4 & 2 \end{bmatrix}.$$

Solution:

Let $A = \begin{bmatrix} 1 & -1 & 5 \\ 1 & 1 & 1 \\ 3 & -4 & 2 \end{bmatrix}$. Then, the cofactors of A are

$$\begin{aligned}
A_{1\,1} &= (-1)^{1+1}\begin{vmatrix} 1 & 1 \\ -4 & 2 \end{vmatrix} = 2-(-4) = 6, \\
A_{1\,2} &= (-1)^{1+2}\begin{vmatrix} 1 & 1 \\ 3 & 2 \end{vmatrix} = -[2-3] = 1, \\
A_{1\,3} &= (-1)^{1+3}\begin{vmatrix} 1 & 1 \\ 3 & -4 \end{vmatrix} = -4-3 = -7, \\
A_{2\,1} &= (-1)^{2+1}\begin{vmatrix} -1 & 5 \\ -4 & 2 \end{vmatrix} = -[-2-(-20)] = -18, \\
A_{2\,2} &= (-1)^{2+2}\begin{vmatrix} 1 & 5 \\ 3 & 2 \end{vmatrix} = 2-15 = -13, \\
A_{2\,3} &= (-1)^{2+3}\begin{vmatrix} 1 & -1 \\ 3 & -4 \end{vmatrix} = -[-4-(-3)] = 1, \\
A_{3\,1} &= (-1)^{3+1}\begin{vmatrix} -1 & 5 \\ 1 & 1 \end{vmatrix} = -1-5 = -6, \\
A_{3\,2} &= (-1)^{3+2}\begin{vmatrix} 1 & 5 \\ 1 & 1 \end{vmatrix} = -[1-5] = 4, \\
\text{and}\quad A_{3\,3} &= (-1)^{3+3}\begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 1-(-1) = 2.
\end{aligned}$$

Alternately, if you can do the correct calculations in your head, you can write each cofactor of A as a subscript of the corresponding entry of A, as follows:

$$\begin{bmatrix} 1_{(6)} & -1_{(1)} & 5_{(-7)} \\ 1_{(-18)} & 1_{(-13)} & 1_{(1)} \\ 3_{(-6)} & -4_{(4)} & 2_{(2)} \end{bmatrix}.$$

Using cofactor expansion along any row or column (say, the second row), we find $|A| = (1)(-18)+(1)(-13)+(1)(1) = -30$. Since $|A| \neq 0$, A does indeed have an inverse. To find it, we take the matrix of the cofactors,

$$\begin{bmatrix} A_{1\,1} & A_{1\,2} & A_{1\,3} \\ A_{2\,1} & A_{2\,2} & A_{2\,3} \\ A_{3\,1} & A_{3\,2} & A_{3\,3} \end{bmatrix} = \begin{bmatrix} 6 & 1 & -7 \\ -18 & -13 & 1 \\ -6 & 4 & 2 \end{bmatrix},$$

compute its transpose to find the adjoint of A,

$$\operatorname{adj}(A) = \begin{bmatrix} 6 & 1 & -7 \\ -18 & -13 & 1 \\ -6 & 4 & 2 \end{bmatrix}^t = \begin{bmatrix} 6 & -18 & -6 \\ 1 & -13 & 4 \\ -7 & 1 & 2 \end{bmatrix},$$

and finally multiply the adjoint of A by $\dfrac{1}{|A|}$ to obtain

$$A^{-1} = \frac{1}{-30}\begin{bmatrix} 6 & -18 & -6 \\ 1 & -13 & 4 \\ -7 & 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{6}{-30} & \frac{-18}{-30} & \frac{-6}{-30} \\ \frac{1}{-30} & \frac{-13}{-30} & \frac{4}{-30} \\ \frac{-7}{-30} & \frac{1}{-30} & \frac{2}{-30} \end{bmatrix} = \begin{bmatrix} -\frac{1}{5} & \frac{3}{5} & \frac{1}{5} \\ -\frac{1}{30} & \frac{13}{30} & -\frac{2}{15} \\ \frac{7}{30} & -\frac{1}{30} & -\frac{1}{15} \end{bmatrix}.$$

$$\begin{bmatrix} 1 & -2 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 0 & 0 & 2 & -7 \\ 0 & 0 & 3 & 0 \end{bmatrix}.$$

<u>Solution</u>:

Let $B = \begin{bmatrix} 1 & -2 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ 0 & 0 & 2 & -7 \\ 0 & 0 & 3 & 0 \end{bmatrix}$. Then, the cofactors of B are

$$B_{1\,1} = +\begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & -7 \\ 0 & 3 & 0 \end{vmatrix} = +\left(+(1)\begin{vmatrix} 2 & -7 \\ 3 & 0 \end{vmatrix}\right) = 21,$$

$$B_{1\,2} = -\begin{vmatrix} 5 & 0 & 0 \\ 0 & 2 & -7 \\ 0 & 3 & 0 \end{vmatrix} = -\left(+(5)\begin{vmatrix} 2 & -7 \\ 3 & 0 \end{vmatrix}\right) = -5(21) = -105,$$

$$B_{1\,3} = +\begin{vmatrix} 5 & 1 & 0 \\ 0 & 0 & -7 \\ 0 & 0 & 0 \end{vmatrix} = +(0) = 0,$$

$$B_{1\,4} = -\begin{vmatrix} 5 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 3 \end{vmatrix} = -\left(+(5)\begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix}\right) = -5(0) = 0,$$

$$B_{2\,1} = -\begin{vmatrix} -2 & 0 & 0 \\ 0 & 2 & -7 \\ 0 & 3 & 0 \end{vmatrix} = -\left(+(-2)\begin{vmatrix} 2 & -7 \\ 3 & 0 \end{vmatrix}\right) = 2(21) = 42,$$

$$B_{2\,2} = +\begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & -7 \\ 0 & 3 & 0 \end{vmatrix} = +\left(+(1)\begin{vmatrix} 2 & -7 \\ 3 & 0 \end{vmatrix}\right) = 21,$$

$$B_{2\,3} = -\begin{vmatrix} 1 & -2 & 0 \\ 0 & 0 & -7 \\ 0 & 0 & 0 \end{vmatrix} = -(0) = 0,$$

$$B_{2\,4} = +\begin{vmatrix} 1 & -2 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 3 \end{vmatrix} = +\left(+(1)\begin{vmatrix} 0 & 2 \\ 0 & 3 \end{vmatrix}\right) = +(0) = 0,$$

$$
\begin{aligned}
B_{3\,1} &= +\begin{vmatrix} -2 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 3 & 0 \end{vmatrix} = +\left(+(-2)\begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix}\right) = -2(0) = 0, \\
B_{3\,2} &= -\begin{vmatrix} 1 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 3 & 0 \end{vmatrix} = -\left(+(1)\begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix}\right) = -(0) = 0, \\
B_{3\,3} &= +\begin{vmatrix} 1 & -2 & 0 \\ 5 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix} = +(0) = 0, \\
B_{3\,4} &= -\begin{vmatrix} 1 & -2 & 0 \\ 5 & 1 & 0 \\ 0 & 0 & 3 \end{vmatrix} = -\left(+(3)\begin{vmatrix} 1 & -2 \\ 5 & 1 \end{vmatrix}\right) = -3(11) = -33, \\
B_{4\,1} &= -\begin{vmatrix} -2 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 2 & -7 \end{vmatrix} = -\left(+(-2)\begin{vmatrix} 0 & 0 \\ 2 & -7 \end{vmatrix}\right) = 2(0) = 0, \\
B_{4\,2} &= +\begin{vmatrix} 1 & 0 & 0 \\ 5 & 0 & 0 \\ 0 & 2 & -7 \end{vmatrix} = +\left(+(1)\begin{vmatrix} 0 & 0 \\ 2 & -7 \end{vmatrix}\right) = +(0) = 0, \\
B_{4\,3} &= -\begin{vmatrix} 1 & -2 & 0 \\ 5 & 1 & 0 \\ 0 & 0 & -7 \end{vmatrix} = -\left(+(-7)\begin{vmatrix} 1 & -2 \\ 5 & 1 \end{vmatrix}\right) = 7(11) = 77, \\
\text{and}\quad B_{4\,4} &= +\begin{vmatrix} 1 & -2 & 0 \\ 5 & 1 & 0 \\ 0 & 0 & 2 \end{vmatrix} = +\left(+(2)\begin{vmatrix} 1 & -2 \\ 5 & 1 \end{vmatrix}\right) = 2(11) = 22.
\end{aligned}
$$

So, writing each cofactor of B as a subscript of the corresponding entry of B yields

$$
\begin{bmatrix} 1_{(21)} & -2_{(-105)} & 0_{(0)} & 0_{(0)} \\ 5_{(42)} & 1_{(21)} & 0_{(0)} & 0_{(0)} \\ 0_{(0)} & 0_{(0)} & 2_{(0)} & -7_{(-33)} \\ 0_{(0)} & 0_{(0)} & 3_{(77)} & 0_{(22)} \end{bmatrix}.
$$

Like before, we use cofactor expansion along any row or column (say, the fourth row) to find $|B| = (3)(77) = 231$. Since $|B| \neq 0$, B has an inverse. To find it, we take the matrix of the cofactors,

$$\begin{bmatrix} B_{1\,1} & B_{1\,2} & B_{1\,3} & B_{1\,4} \\ B_{2\,1} & B_{2\,2} & B_{2\,3} & B_{2\,4} \\ B_{3\,1} & B_{3\,2} & B_{3\,3} & B_{3\,4} \\ B_{4\,1} & B_{4\,2} & B_{4\,3} & B_{4\,4} \end{bmatrix} = \begin{bmatrix} 21 & -105 & 0 & 0 \\ 42 & 21 & 0 & 0 \\ 0 & 0 & 0 & -33 \\ 0 & 0 & 77 & 22 \end{bmatrix},$$

compute its transpose to find the adjoint of B,

$$\text{adj}(B) = \begin{bmatrix} 21 & -105 & 0 & 0 \\ 42 & 21 & 0 & 0 \\ 0 & 0 & 0 & -33 \\ 0 & 0 & 77 & 22 \end{bmatrix}^t = \begin{bmatrix} 21 & 42 & 0 & 0 \\ -105 & 21 & 0 & 0 \\ 0 & 0 & 0 & 77 \\ 0 & 0 & -33 & 22 \end{bmatrix},$$

and lastly multiply the adjoint of B by $\frac{1}{|B|}$ to obtain

$$B^{-1} = \frac{1}{231} \begin{bmatrix} 21 & 42 & 0 & 0 \\ -105 & 21 & 0 & 0 \\ 0 & 0 & 0 & 77 \\ 0 & 0 & -33 & 22 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{21}{231} & \frac{42}{231} & \frac{0}{231} & \frac{0}{231} \\ \frac{-105}{231} & \frac{21}{231} & \frac{0}{231} & \frac{0}{231} \\ \frac{0}{231} & \frac{0}{231} & \frac{0}{231} & \frac{77}{231} \\ \frac{0}{231} & \frac{0}{231} & \frac{-33}{231} & \frac{22}{231} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{11} & \frac{2}{11} & 0 & 0 \\ -\frac{5}{11} & \frac{1}{11} & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & -\frac{1}{7} & \frac{2}{21} \end{bmatrix}.$$

4. Find all values of a, b, c and d for which the following matrix is singular (a square matrix A is called singular if $|A| = 0$):

$$\begin{bmatrix} 2a & -1 & 1 & 1 \\ b & 2 & 1 & 1 \\ 0 & 0 & c & d \\ 0 & 0 & 1 & -4 \end{bmatrix}.$$

Solution:

We first need to find the determinant of this matrix, as follows:

$$\begin{aligned}
&\begin{vmatrix} 2a & -1 & 1 & 1 \\ b & 2 & 1 & 1 \\ 0 & 0 & c & d \\ 0 & 0 & 1 & -4 \end{vmatrix} \\
= &\left|\begin{array}{cc|cc} 2a & -1 & 1 & 1 \\ b & 2 & 1 & 1 \\ \hline 0 & 0 & c & d \\ 0 & 0 & 1 & -4 \end{array}\right| \\
= &\begin{vmatrix} 2a & -1 \\ b & 2 \end{vmatrix} \cdot \begin{vmatrix} c & d \\ 1 & -4 \end{vmatrix} \\
= &\ (4a+b)\cdot(-4c-d) \\
= &\ -(4a+b)(4c+d).
\end{aligned}$$

By observation, we see that the determinant of this matrix is 0 if $4a+b=0$ or $4c+d=0$. Therefore, this matrix is singular for the following values of a, b, c, and d:

$$\begin{cases} a = -\frac{1}{4}s \\ b = s \\ c = -\frac{1}{4}t \\ d = t \end{cases}.$$

5. Prove or disprove each of the following statements:

(a) $|A+B| = |A| + |B|$, for all (square) matrices A and B.

Solution: This statement is FALSE.

Let $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

Then, $|A| = 0$, and $|B| = 0$, so $|A| + |B| = 0 + 0 = 0$. However, $A + B = I_2$, so $|A+B| = |I_2| = 1$.

Hence, $|A+B| \neq |A| + |B|$ for all square matrices A and B.

(b) $|2A| = 8|A|$, for all 3 by 3 matrices A.

Solution: This statement is TRUE.

Let A be any 3×3 matrix. Then,

$$\begin{aligned}
& |2A| \\
= \; & |2\,(I_3 A)| \\
= \; & |(2I_3)\,A| \\
= \; & |2I_3| \cdot |A| \\
= \; & \left| 2 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| \cdot |A| \\
= \; & \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix} \cdot |A| \\
= \; & +(2) \begin{vmatrix} 2 & 0 \\ 0 & 2 \end{vmatrix} \cdot |A| \\
= \; & 2\,[4 - 0]\,|A| \\
= \; & 8\,|A|, \quad \text{as required.}
\end{aligned}$$

(c) $\left|(AB)^{-1}\right| = \frac{1}{|A||B|}$, for all nonsingular (square) matrices A and B.

Solution: This statement is TRUE.

Let A and B be nonsingular matrices of size $n \times n$. Then, $|A| \neq 0$ and $|B| \neq 0$, so A and B are invertible i.e. both A^{-1} and B^{-1} exist. Moreover,

$$
\begin{aligned}
& B^{-1}A^{-1}AB = I_n \\
\Longrightarrow \quad & (AB)^{-1}(AB) = I_n \\
\Longrightarrow \quad & \left|(AB)^{-1}(AB)\right| = |I_n| \\
\Longrightarrow \quad & \left|(AB)^{-1}\right| \cdot |AB| = 1 \\
\Longrightarrow \quad & \left|(AB)^{-1}\right| \cdot (|A| \cdot |B|) = 1 \\
\Longrightarrow \quad & \left|(AB)^{-1}\right| = \tfrac{1}{|A||B|}, \quad \text{as required.}
\end{aligned}
$$

6. Use the Cramer's rule to solve the following system of linear equations for any values of $a \neq 0$ and $b \neq 2$:

$$
\begin{array}{rcrcrcr}
x & + & ay & + & bz & = & 10 \\
x & + & ay & + & 2z & = & 2 \\
2x & + & ay & + & 3z & = & 5
\end{array}
$$

Solution:

Let $A = \begin{bmatrix} 1 & a & b \\ 1 & a & 2 \\ 2 & a & 3 \end{bmatrix}$, and for $i = 1,2,3$, let A_i be the matrix obtained by replacing column i of A with the column $B = \begin{bmatrix} 10 \\ 2 \\ 5 \end{bmatrix}$.

Then, $|A| = \begin{vmatrix} 1 & a & b \\ 1 & a & 2 \\ 2 & a & 3 \end{vmatrix}$

$$\underset{-\text{R1}+\text{R2}}{=} \begin{vmatrix} 1 & a & b \\ 0 & 0 & 2-b \\ 2 & a & 3 \end{vmatrix}$$

$$= -(2-b)\begin{vmatrix} 1 & a \\ 2 & a \end{vmatrix}$$

$$= -(2-b)(-a)$$

$$= a(2-b),$$

$$|A_1| = \begin{vmatrix} 10 & a & b \\ 2 & a & 2 \\ 5 & a & 3 \end{vmatrix}$$

$$\underset{\substack{-\text{R2}+\text{R1} \\ -\text{R2}+\text{R3}}}{=} \begin{vmatrix} 8 & 0 & b-2 \\ 2 & a & 2 \\ 3 & 0 & 1 \end{vmatrix}$$

$$= +(a)\begin{vmatrix} 8 & b-2 \\ 3 & 1 \end{vmatrix}$$

$$= a(8-3(b-2))$$

$$= a(14-3b),$$

$$|A_2| = \begin{vmatrix} 1 & 10 & b \\ 1 & 2 & 2 \\ 2 & 5 & 3 \end{vmatrix}$$

$$\underset{\substack{-\text{R2}+\text{R1} \\ -2\text{R2}+\text{R3}}}{=} \begin{vmatrix} 0 & 8 & b-2 \\ 1 & 2 & 2 \\ 0 & 1 & -1 \end{vmatrix}$$

$$= -(1)\begin{vmatrix} 8 & b-2 \\ 1 & -1 \end{vmatrix}$$

$$= -(-8-(b-2))$$

$$= b+6,$$

$$\text{and}\quad |A_3| = \begin{vmatrix} 1 & a & 10 \\ 1 & a & 2 \\ 2 & a & 5 \end{vmatrix}$$

$$\underset{-\mathsf{R1}+\mathsf{R2}}{=} \begin{vmatrix} 1 & a & 10 \\ 0 & 0 & -8 \\ 2 & a & 5 \end{vmatrix}$$

$$= -(-8)\begin{vmatrix} 1 & a \\ 2 & a \end{vmatrix}$$

$$= 8(-a)$$

$$= -8a.$$

By Cramer's rule, the unique solution for the system is

$$\begin{cases} x = \frac{|A_1|}{|A|} = \frac{a(14-3b)}{a(2-b)} \\ y = \frac{|A_2|}{|A|} = \frac{b+6}{a(2-b)} \\ z = \frac{|A_3|}{|A|} = \frac{-8a}{a(2-b)}, \end{cases}$$

$$\text{or}\quad \begin{cases} x = \frac{3b-14}{b-2} \\ y = -\frac{b+6}{a(b-2)} \\ z = \frac{8}{b-2}. \end{cases}$$