University of Lethbridge • Department of Mathematics and Computer Science
Calculus I • Math 2560 • February 27, 1999
Midterm Examination

Name:______________________________ ID #:______________

Date: Saturday February 27, 1999 Time: 10:00–12:00

Instructor: H. Kharaghani

1. (a) (10) Sketch a (rough) graph of $y = \cos x$ and show that it is not a one-to-one function, but $y = \cos x, 0 \leq x \leq \pi$, is one-to-one. Show that $(\cos^{-1} x)' = \dfrac{-1}{\sqrt{1-x^2}}$ and sketch a (rough) graph of $y = \cos^{-1} x$.

   (b) (7) Let $y = \sin^{-1} x + \cos^{-1} x$. Show that $y' = 0$ for all x in $(-1, 1)$. Conclude that $\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$ for all x in $[-1, 1]$.

2. (5,5,5,5) Find y' for each of the following functions:

(a) $$y = \ln \frac{(x^2+1)\sqrt[5]{x^4+x^2+1}}{x^2(\sin x + 5)}$$

(b) $y = \cos^{-1}(\dfrac{\sin x}{3+\sin x})$.

(c) $y = e^{\cos^{-1}(x^3)}$.

(d) $y = (\sin x + 2)^{\cos x}$.

3. (6,6,6) Evaluate the following limits:

(a) $$\lim_{x\to 0} \frac{\tan x - x}{x^3}$$

(b) $$\lim_{x\to 0^+} (\cot x)^{\sin x}$$

(c) $$\lim_{x\to 0^+} (\frac{1}{x} - \frac{1}{\sin x})$$

4. (9) Assuming that $\alpha > 0$, show that $\lim\limits_{x\to\infty} \dfrac{x^\alpha}{(\ln x)^3} = \infty$ and thus conclude that for large values of x, $x^\alpha \geq 1000(\ln x)^3$.

5. (a) (6) Find the area of the region bounded by the curves $y = x^3 + x^2$, $y = 2x^2$ from $x = -1$ to $x = 2$.

(b) (6) Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the curves $y = \cos x$, $y = 0$, $x = 0$, $x = \frac{\pi}{2}$ about $y = 2$.

(c) (6) Set up, but do not evaluate, an integral for the volume of the solid obtained by rotating the region bounded by the curves $x = 4 - y^2$, $x = 8 - 2y^2$, about $y = 5$, using the cylindrical shells method.

(d) (6) A circular swimming pool has a diameter of 20 ft, the sides are 5 ft high and the depth of the water is 4 ft. How much work is required to pump all the water out over to 1 ft above the sides.

6. (a) (6) Evaluate the integral $\int \frac{\sinh x}{1 + \cosh x}\, dx$.

(b) (6) Evaluate the following limit:

$$\lim_{x \to \infty} \frac{\sinh x}{e^x}$$

(c) Bonus (8) Use the definition of the derivative to prove that:

$$\lim_{x \to 0} \frac{\ln(1 + x)}{x} = 1$$