Some new orthogonal designs in orders 32 and 40

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Abstract

A result of Robinson states that no $OD(n; 1, 1, 1, 1, 1, n-5)$ exists for $n > 40$. We complement this result by showing the existence of $OD(n; 1, 1, 1, 1, 1, n-5)$ for $n = 32, 40$. This includes a resolution to an old open problem regarding orthogonal designs of order 32 as well. We also obtain a number of new orthogonal designs of order 32.

Keywords: orthogonal designs, exhaustive search.
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1 Introduction

An orthogonal design D of order n and type $(u_1, u_2, \ldots, u_k)$ denoted $OD(n; u_1, u_2, \ldots, u_k)$ is a matrix of order n with entries in the set $\{0, \pm x_1, \pm x_2, \ldots, \pm x_k\}$, x_i's commuting variables, such that

$$DD^t = \sum_{i=1}^{k} (u_i x_i^2) I_n,$$

where I_n is the identity matrix of order n. It is known that the number of variables in an orthogonal design never exceeds $\rho(n)$, the Radon number of n, where for $n = 2^{4c+d}m$, m odd, $0 \leq d < 4$, $\rho(n) = 8c + 2^d$.

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Peter Robinson showed that there is no $OD(n; 1, 1, 1, 1, 1, n-5)$ for $n > 40$, see [4, 5]. The existence of these orthogonal designs thus is only possible for $n = 8, 16, 24, 32, 40$. The case of $n = 8$ has been known for a long time. Robinson [2, 3] demonstrated their existence for $n = 16, 24$, but the two cases of $n = 32, 40$ remained open since he wrote his Ph. D thesis in 1971. We resolve these two cases with the following theorem.

Theorem 1 *An $OD(n; 1, 1, 1, 1, 1, n-5)$ exists if and only if $n = 8, 16, 24, 32, 40$.*

Note that, as a corollary, this also resolves the two remaining open questions regarding the existence of orthogonal designs of order 32 in 5 and 6 variables, see [1, p394]. Specifically, we have the following corollary.

Corollary 1 *All 5-tuples and all full 6-tuples are the types of orthogonal designs in order 32.*

In addition to this, we did an exhaustive computer search for full $OD(32; 1, 1, 1, 1, 1, u_1, \ldots, u_k)$. Our results can be summarized in the following theorem.

Theorem 2 *There is a full $OD(32; 1, 1, 1, 1, 1, u_1, \ldots, u_k)$ if and only if $(u_1, \ldots, u_k) = (9, 9, 9)$, $(9, 18)$, $(12, 15)$, (27).*

2 The structure of $OD(n; 1, 1, 1, 1, 1)$

We first need to know the general structure of an $OD(n; 1, 1, 1, 1, 1)$ for a given n, n a multiple of 8. To do this, we wrote a simple computer program and it was shown that such an orthogonal design up to equivalence has to have blocks of size 8 with all off diagonal blocks being the zero matrix and the diagonal blocks are all $OD(8; 1, 1, 1, 1, 1)$. There are 10 different $OD(8; 1, 1, 1, 1, 1)$ up to negations and permutations of rows and columns. These orthogonal designs in variables a, b, c, d, e are shown below. The parameter x can be 1 or -1 in any of the matrices. For convenience, $-a$ is denoted by $\bar{a}$, etc.

$$D_1 = \begin{bmatrix} a & b & c & 0 & xd & 0 & 0 & \bar{e} \\ \bar{b} & a & 0 & c & 0 & x\bar{d} & e & 0 \\ \bar{c} & 0 & a & \bar{b} & 0 & \bar{e} & x\bar{d} & 0 \\ 0 & \bar{c} & b & a & e & 0 & 0 & xd \\ x\bar{d} & 0 & 0 & \bar{e} & a & b & c & 0 \\ 0 & xd & e & 0 & \bar{b} & a & 0 & c \\ 0 & \bar{e} & xd & 0 & \bar{c} & 0 & a & \bar{b} \\ e & 0 & 0 & x\bar{d} & 0 & \bar{c} & b & a \end{bmatrix}, D_2 = \begin{bmatrix} a & b & c & 0 & xd & 0 & e & 0 \\ \bar{b} & a & 0 & c & 0 & x\bar{d} & 0 & e \\ \bar{c} & 0 & a & \bar{b} & e & 0 & x\bar{d} & 0 \\ 0 & \bar{c} & b & a & 0 & e & 0 & xd \\ x\bar{d} & 0 & \bar{e} & 0 & a & b & c & 0 \\ 0 & xd & 0 & \bar{e} & \bar{b} & a & 0 & c \\ \bar{e} & 0 & xd & 0 & \bar{c} & 0 & a & \bar{b} \\ 0 & \bar{e} & 0 & x\bar{d} & 0 & \bar{c} & b & a \end{bmatrix},$$

$$D_3 = \begin{bmatrix} a & b & c & 0 & xd & e & 0 & 0 \\ \bar{b} & a & 0 & c & e & x\bar{d} & 0 & 0 \\ \bar{c} & 0 & a & \bar{b} & 0 & 0 & x\bar{d} & \bar{e} \\ 0 & \bar{c} & b & a & 0 & 0 & \bar{e} & xd \\ x\bar{d} & \bar{e} & 0 & 0 & a & b & c & 0 \\ \bar{e} & xd & 0 & 0 & \bar{b} & a & 0 & c \\ 0 & 0 & xd & e & \bar{c} & 0 & a & \bar{b} \\ 0 & 0 & e & x\bar{d} & 0 & \bar{c} & b & a \end{bmatrix}, D_4 = \begin{bmatrix} a & b & c & x\bar{d} & e & 0 & 0 & 0 \\ \bar{b} & a & xd & c & 0 & \bar{e} & 0 & 0 \\ \bar{c} & x\bar{d} & a & \bar{b} & 0 & 0 & \bar{e} & 0 \\ xd & \bar{c} & b & a & 0 & 0 & 0 & e \\ \bar{e} & 0 & 0 & 0 & a & b & c & xd \\ 0 & e & 0 & 0 & \bar{b} & a & x\bar{d} & c \\ 0 & 0 & e & 0 & \bar{c} & xd & a & \bar{b} \\ 0 & 0 & 0 & \bar{e} & x\bar{d} & \bar{c} & b & a \end{bmatrix},$$

$$D_5 = \begin{bmatrix} a & b & c & x\bar{e} & d & 0 & 0 & 0 \\ \bar{b} & a & xe & c & 0 & \bar{d} & 0 & 0 \\ \bar{c} & x\bar{e} & a & \bar{b} & 0 & 0 & \bar{d} & 0 \\ xe & \bar{c} & b & a & 0 & 0 & 0 & d \\ \bar{d} & 0 & 0 & 0 & a & b & c & xe \\ 0 & d & 0 & 0 & \bar{b} & a & x\bar{e} & c \\ 0 & 0 & d & 0 & \bar{c} & xe & a & \bar{b} \\ 0 & 0 & 0 & \bar{d} & x\bar{e} & \bar{c} & b & a \end{bmatrix}.$$

3 An exhaustive search

In this section we describe our method of exhaustive search to find all possible full $OD(32; 1, 1, 1, 1, 1, u_1, \ldots, u_k)$. In order to do this, we first had to construct all possible $OD(32; 1, 1, 1, 1, 1)$. From the results of the previous section, any such orthogonal design D may be taken to have D_i as its diagonal blocks. We then replaced the zeros of D by different variables to find an $OD(32; 1, 1, 1, 1, 1, u_1, \ldots, u_k)$. There are a total of 864 zeros in D. However, by paying attention to the orthogonality of the new matrix, we need only 36 different variables satisfying a system of 24 equations.

Remark 1 In any full $OD(n; 1, 1, 1, 1, 1, n-5)$ no two diagonal blocks can come from the same D_i. This follows from an argument similar to that of Robinson [4]. (Note that our general approach is quite different from that of Robinson, but this particular argument is similar to his line of reasoning and for the sake of brevity we are not duplicating it here.) This shows the reason that why no $OD(n; 1, 1, 1, 1, 1, n-5)$ may exist for $n > 40$.

There are altogether $\binom{5}{4}2^4 = 80$ systems, each containing 24 equations in 36 variables, to be solved. It is easy to see that, for each of the systems, there are a maximum of $(2k)^{36}$ cases to be checked in order to find all $OD(32; 1, 1, 1, 1, 1, u_1, \ldots, u_k)$. Obviously $k = 1$ is the only feasible case. Our search in this case turned in a total of $2^{15} = 32768$ solutions corresponding to each of 80 systems. This was simply done using a backtrack search.

Now we have all $OD(32; 1, 1, 1, 1, 1, 27)$, up to negations and permutations of rows and columns. If there is a full $OD(32; 1, 1, 1, 1, 1, u_1, \ldots, u_k)$, $k > 1$, then by taking $u_1 = u_2 = \cdots = u_k$, one gets an $OD(32; 1, 1, 1, 1, 1, 27)$. Therefore, our data may be used to construct all possible full $OD(32; 1, 1, 1, 1, 1, u_1, \ldots, u_k)$, $k > 1$, by a reversing process. For example, in order to find all possible $OD(32; 1, 1, 1, 1, 1, a, 27 - a)$, let $D_1 = [b_{ij}]$ and $D_2 = [c_{ij}]$ be two $OD(32; 1, 1, 1, 1, 1, 27)$, where, say, the variable x is repeated 27 times in both D_1 and D_2. Define a matrix $D = [d_{ij}]$ by

$$d_{ij} = \begin{cases} \pm y & \text{if } b_{ij} = \pm x \text{ and } c_{ij} = \mp x, \\ \pm z & \text{if } b_{ij} = \pm x \text{ and } c_{ij} = \pm x, \\ b_{ij} & \quad\quad\text{otherwise.} \end{cases}$$

Then D is a candidate for an $OD(32; 1, 1, 1, 1, 1, a, 27 - a)$. If it is orthogonal, then we are done. We examined all 32768×32768 possibilities for (D_1, D_2) in each of the 80 systems and found only orthogonal designs of type $(1, 1, 1, 1, 1, 9, 18)$ and $(1, 1, 1, 1, 1, 12, 15)$. Applying a similar method, we search for all full $OD(32; 1, 1, 1, 1, 1, u_1, \ldots, u_k)$, $k > 2$. To see this, we first assume $k = 3$, and $u_1 \neq u_2 \neq u_3$. Since the different collapsing of the variables which are repeated u_1, u_2 and u_3 times gives three different full orthogonal designs in 7 variables and there are only two of these of above type, we conclude that at least two of u_is must be the same. The only possibility then is $u_1 = u_2 = u_3 = 9$. It is now clear that $k \leq 3$ and the only possible $OD(32; 1, 1, 1, 1, 1, u_1, \ldots, u_k)$, $k > 2$ is $OD(32; 1, 1, 1, 1, 1, 9, 9, 9)$. Our search was successful and an $OD(32; 1, 1, 1, 1, 1, 9, 9, 9)$ was found.

A similar approach may be applied to search for the remaining full orthogonal designs containing five variables each repeated once, namely, an $OD(40; 1, 1, 1, 1, 1, u_1, \ldots, u_k)$. In this case we get a system of 40 equations in 55 variables. Finding all the solutions through an exhaustive search is not feasible. So we only searched for one solution and an $OD(40; 1, 1, 1, 1, 1, 35)$ was found. Perhaps it is appropriate to echo Robinson's statement that "the combinatorial structure (of orthogonal designs) is much more restrictive than previously thought (to be)".

The new orthogonal designs; $OD(32; 1, 1, 1, 1, 1, 12, 15)$, $OD(32; 1, 1, 1, 1, 1, 9, 9, 9)$, and $OD(40; 1, 1, 1, 1, 1, 35)$ are given below.

$$
\left[\begin{array}{cccccccccccccccccccccccccccccccc}
a & b & c & x & d & y & x & \bar{e} & \bar{x} & x & \bar{y} & \bar{y} & \bar{x} & x & y & y & x & x & x & x & \bar{y} & y & \bar{y} & y & \bar{y} & \bar{x} & \bar{y} & \bar{y} & \bar{y} & x & y & \bar{y} \\
\bar{b} & a & \bar{x} & c & y & \bar{d} & e & x & x & x & y & \bar{y} & x & x & \bar{y} & y & x & \bar{x} & \bar{x} & x & y & y & \bar{y} & \bar{y} & \bar{x} & y & y & \bar{y} & x & y & y & y \\
\bar{c} & x & a & \bar{b} & x & \bar{e} & \bar{d} & \bar{y} & \bar{y} & \bar{y} & x & \bar{x} & y & y & x & \bar{x} & x & x & \bar{x} & \bar{x} & \bar{y} & y & y & \bar{y} & \bar{y} & \bar{y} & y & x & y & \bar{y} & y & \bar{x} \\
\bar{x} & \bar{c} & b & a & e & x & \bar{y} & d & y & \bar{y} & \bar{x} & \bar{x} & \bar{y} & y & \bar{x} & \bar{x} & \bar{x} & x & \bar{x} & x & \bar{y} & \bar{y} & \bar{y} & \bar{y} & y & \bar{y} & x & \bar{y} & y & y & \bar{x} & \bar{y} \\
\bar{d} & \bar{y} & \bar{x} & \bar{e} & a & b & c & \bar{x} & \bar{x} & \bar{x} & \bar{y} & y & x & x & \bar{y} & y & \bar{y} & \bar{y} & y & y & \bar{x} & x & x & \bar{x} & y & \bar{y} & x & \bar{y} & \bar{y} & \bar{y} & x & y \\
\bar{y} & d & e & \bar{x} & \bar{b} & a & x & c & \bar{x} & x & \bar{y} & \bar{y} & x & \bar{x} & \bar{y} & \bar{y} & \bar{y} & y & \bar{y} & y & x & x & x & x & \bar{y} & \bar{y} & y & x & \bar{y} & y & \bar{y} & x \\
\bar{x} & \bar{e} & d & y & \bar{c} & \bar{x} & a & \bar{b} & \bar{y} & y & x & x & \bar{y} & y & \bar{x} & \bar{x} & y & y & y & y & x & \bar{x} & x & \bar{x} & x & \bar{y} & \bar{y} & y & x & y & y & y \\
e & \bar{x} & y & \bar{d} & x & \bar{c} & b & a & \bar{y} & \bar{y} & x & \bar{x} & \bar{y} & \bar{y} & \bar{x} & x & \bar{y} & y & y & \bar{y} & x & x & \bar{x} & \bar{x} & y & x & y & y & \bar{y} & x & y & \bar{y} \\
x & \bar{x} & y & \bar{y} & x & x & y & y & a & b & c & \bar{x} & d & \bar{x} & e & \bar{y} & \bar{y} & \bar{y} & y & \bar{y} & \bar{y} & \bar{x} & x & \bar{y} & \bar{y} & \bar{x} & \bar{y} & \bar{x} & x & y & x & y \\
\bar{x} & \bar{x} & y & y & x & \bar{x} & \bar{y} & y & \bar{b} & a & x & c & \bar{x} & \bar{d} & y & e & \bar{y} & y & y & y & \bar{x} & y & y & x & \bar{x} & y & x & \bar{y} & y & \bar{x} & \bar{y} & x \\
y & \bar{y} & \bar{x} & x & y & y & \bar{x} & \bar{x} & \bar{c} & \bar{x} & a & \bar{b} & e & \bar{y} & \bar{d} & x & y & \bar{y} & y & y & x & \bar{y} & y & x & \bar{y} & \bar{x} & y & x & x & y & \bar{x} & \bar{y} \\
y & y & x & x & \bar{y} & y & \bar{x} & x & x & \bar{c} & b & a & y & e & x & d & y & y & y & \bar{y} & y & x & x & \bar{y} & x & \bar{y} & x & \bar{y} & \bar{y} & x & \bar{y} & x \\
x & \bar{x} & \bar{y} & y & \bar{x} & \bar{x} & y & y & \bar{d} & x & \bar{e} & \bar{y} & a & b & c & x & \bar{y} & x & \bar{x} & \bar{y} & y & \bar{y} & y & y & x & \bar{y} & x & \bar{y} & y & \bar{x} & y & \bar{x} \\
\bar{x} & \bar{x} & \bar{y} & \bar{y} & \bar{x} & x & \bar{y} & y & x & d & y & \bar{e} & \bar{b} & a & \bar{x} & c & x & y & y & \bar{x} & \bar{y} & \bar{y} & \bar{y} & y & \bar{y} & \bar{x} & y & x & \bar{x} & \bar{y} & x & y \\
\bar{y} & y & \bar{x} & x & y & y & x & x & \bar{e} & \bar{y} & d & \bar{x} & \bar{c} & x & a & \bar{b} & \bar{x} & \bar{y} & y & \bar{x} & y & y & \bar{y} & y & x & \bar{y} & \bar{x} & y & y & \bar{x} & \bar{y} & x \\
\bar{y} & \bar{y} & x & x & \bar{y} & y & x & \bar{x} & y & \bar{e} & \bar{x} & \bar{d} & \bar{x} & \bar{c} & b & a & y & \bar{x} & \bar{x} & \bar{y} & \bar{y} & y & y & y & y & x & y & x & x & y & x & y \\
\bar{x} & \bar{x} & \bar{x} & x & y & y & \bar{y} & y & y & y & \bar{y} & \bar{y} & y & \bar{x} & x & \bar{y} & a & b & c & x & d & e & x & \bar{y} & y & x & \bar{x} & y & \bar{x} & \bar{y} & y & \bar{x} \\
\bar{x} & x & \bar{x} & \bar{x} & y & \bar{y} & \bar{y} & \bar{y} & y & \bar{y} & y & \bar{y} & \bar{x} & \bar{y} & y & x & \bar{b} & a & \bar{x} & c & e & \bar{d} & y & x & x & \bar{y} & \bar{y} & \bar{x} & \bar{y} & x & x & y \\
\bar{x} & x & x & x & \bar{y} & y & \bar{y} & \bar{y} & \bar{y} & \bar{y} & \bar{y} & \bar{y} & x & \bar{y} & \bar{y} & x & \bar{c} & x & a & \bar{b} & x & \bar{y} & \bar{d} & \bar{e} & \bar{x} & y & \bar{y} & \bar{x} & y & \bar{x} & x & y \\
\bar{x} & \bar{x} & x & \bar{x} & \bar{y} & \bar{y} & \bar{y} & y & y & \bar{y} & \bar{y} & y & y & x & x & y & \bar{x} & \bar{c} & b & a & y & x & \bar{e} & d & \bar{y} & \bar{x} & \bar{x} & y & x & y & y & \bar{x} \\
y & \bar{y} & y & y & x & \bar{x} & \bar{x} & \bar{x} & y & x & \bar{x} & \bar{y} & \bar{y} & y & \bar{y} & y & \bar{d} & \bar{e} & \bar{x} & \bar{y} & a & b & c & \bar{x} & \bar{y} & \bar{x} & \bar{x} & y & \bar{x} & \bar{y} & \bar{y} & x \\
\bar{y} & \bar{y} & \bar{y} & y & \bar{x} & \bar{x} & x & \bar{x} & x & \bar{y} & y & \bar{x} & y & y & \bar{y} & \bar{y} & \bar{e} & d & y & \bar{x} & \bar{b} & a & x & c & \bar{x} & y & \bar{y} & \bar{x} & \bar{y} & x & \bar{x} & \bar{y} \\
y & y & \bar{y} & y & \bar{x} & \bar{x} & \bar{x} & x & \bar{x} & \bar{y} & \bar{y} & \bar{x} & \bar{y} & y & y & \bar{y} & \bar{x} & \bar{y} & d & e & \bar{c} & \bar{x} & a & \bar{b} & \bar{x} & y & y & x & \bar{y} & x & x & y \\
\bar{y} & y & y & y & x & \bar{x} & x & x & y & \bar{x} & \bar{x} & y & \bar{y} & \bar{y} & \bar{y} & \bar{y} & y & \bar{x} & e & \bar{d} & x & \bar{c} & b & a & \bar{y} & \bar{x} & x & \bar{y} & \bar{x} & \bar{y} & y & \bar{x} \\
y & x & y & \bar{y} & \bar{y} & y & \bar{x} & \bar{y} & y & x & y & \bar{x} & \bar{x} & y & \bar{x} & \bar{y} & \bar{y} & \bar{x} & x & y & y & x & x & y & a & b & c & \bar{d} & e & \bar{x} & y & \bar{x} \\
x & \bar{y} & y & y & y & y & y & \bar{x} & x & \bar{y} & x & y & y & x & y & \bar{x} & \bar{x} & y & \bar{y} & x & x & \bar{y} & \bar{y} & x & \bar{b} & a & d & c & \bar{x} & \bar{e} & x & y \\
y & \bar{y} & \bar{y} & \bar{x} & \bar{x} & \bar{y} & y & \bar{y} & y & \bar{x} & \bar{y} & \bar{x} & \bar{x} & \bar{y} & x & \bar{y} & x & y & y & x & x & y & \bar{y} & \bar{x} & \bar{c} & \bar{d} & a & \bar{b} & y & \bar{x} & \bar{e} & x \\
y & y & \bar{x} & y & y & \bar{x} & \bar{y} & \bar{y} & x & y & \bar{x} & y & y & \bar{x} & \bar{y} & \bar{x} & \bar{y} & x & x & \bar{y} & \bar{y} & x & \bar{x} & y & d & \bar{c} & b & a & x & y & x & e \\
y & \bar{x} & \bar{y} & \bar{y} & y & y & \bar{x} & y & \bar{x} & \bar{y} & \bar{x} & y & \bar{y} & x & \bar{y} & \bar{x} & x & y & \bar{y} & \bar{x} & x & y & y & x & \bar{e} & x & \bar{y} & \bar{x} & a & b & c & d \\
\bar{x} & \bar{y} & y & \bar{y} & y & \bar{y} & \bar{y} & \bar{x} & \bar{y} & x & \bar{y} & \bar{x} & x & y & x & \bar{y} & y & \bar{x} & x & \bar{y} & y & \bar{x} & \bar{x} & y & x & e & x & \bar{y} & \bar{b} & a & \bar{d} & c \\
\bar{y} & \bar{y} & \bar{y} & x & \bar{x} & y & \bar{y} & \bar{y} & \bar{x} & y & x & y & \bar{y} & \bar{x} & y & \bar{x} & \bar{y} & \bar{x} & \bar{x} & \bar{y} & y & x & \bar{x} & \bar{y} & \bar{y} & \bar{x} & e & \bar{x} & \bar{c} & d & a & \bar{b} \\
y & \bar{y} & x & y & \bar{y} & \bar{x} & \bar{y} & y & \bar{y} & \bar{x} & y & \bar{x} & x & \bar{y} & \bar{x} & \bar{y} & x & \bar{y} & \bar{y} & x & \bar{x} & y & \bar{y} & x & x & \bar{y} & \bar{x} & \bar{e} & \bar{d} & \bar{c} & b & a
\end{array}\right]
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An $OD(32;1,1,1,1,1,12,15)$

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\left[\begin{array}{cccccccccccccccccccccccccccccccc}
a & b & c & x & d & z & y & \bar{e} & \bar{x} & x & \bar{x} & \bar{x} & \bar{y} & y & y & y & z & y & z & y & \bar{y} & z & \bar{y} & z & \bar{x} & \bar{z} & \bar{z} & \bar{x} & \bar{x} & z & z & \bar{x} \\
\bar{b} & a & \bar{x} & c & z & \bar{d} & e & y & x & x & x & \bar{x} & y & y & \bar{y} & y & y & \bar{z} & \bar{y} & z & z & y & \bar{z} & \bar{y} & \bar{z} & x & x & \bar{z} & z & x & x & z \\
\bar{c} & x & a & \bar{b} & y & \bar{e} & \bar{d} & \bar{z} & \bar{x} & \bar{x} & x & \bar{x} & y & y & y & \bar{y} & z & y & \bar{z} & \bar{y} & \bar{y} & z & y & \bar{z} & \bar{z} & \bar{x} & x & z & z & \bar{x} & x & \bar{z} \\
\bar{x} & \bar{c} & b & a & e & y & \bar{z} & d & x & \bar{x} & \bar{x} & \bar{x} & \bar{y} & y & \bar{y} & \bar{y} & \bar{y} & z & \bar{y} & z & \bar{z} & \bar{y} & \bar{z} & \bar{y} & x & \bar{z} & z & \bar{x} & x & z & \bar{z} & \bar{x} \\
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\end{array}\right]
$$

An $OD(32;1,1,1,1,1,9,9,9)$

$$
\begin{bmatrix}
a & b & c & x & d & x & x & \overline{e} & \overline{x} & x & x & x & \overline{x} & x & \overline{x} & \overline{x} & x & \overline{x} & x & \overline{x} & x & x & x & x & \overline{x} & x & x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & \overline{x} & x \\
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\overline{d} & \overline{x} & \overline{x} & \overline{e} & a & b & c & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & x & x & x & \overline{x} & x & \overline{x} & \overline{x} & x & \overline{x} & \overline{x} & x & x & x & x & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & x & x & x & x & x & x & \overline{x} & \overline{x} & x \\
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\overline{x} & \overline{e} & d & x & \overline{c} & \overline{x} & a & \overline{b} & x & \overline{x} & x & x & x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & x & x & x & x & x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & x & \overline{x} & x & x & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & x \\
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x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & x & x & x & \overline{x} & \overline{x} & x & \overline{x} & \overline{x} & x & x & \overline{x} & x & x & \overline{x} & \overline{x} & \overline{x} & a & b & c & \overline{d} & e & x & \overline{x} & \overline{x} & x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} \\
\overline{x} & \overline{x} & x & \overline{x} & \overline{x} & x & x & x & x & \overline{x} & x & x & x & x & x & \overline{x} & x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & \overline{b} & a & d & c & x & \overline{e} & x & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & \overline{x} & x & x & x \\
\overline{x} & \overline{x} & \overline{x} & x & x & \overline{x} & x & x & x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & \overline{x} & x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & \overline{c} & \overline{d} & a & \overline{b} & \overline{x} & \overline{x} & e & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & x & \overline{x} & x & x \\
x & \overline{x} & x & x & x & x & x & \overline{x} & x & x & \overline{x} & x & x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & x & \overline{x} & x & x & d & \overline{c} & b & a & x & \overline{x} & \overline{x} & e & x & \overline{x} & x & x & x & x & x & \overline{x} \\
x & x & x & \overline{x} & x & \overline{x} & x & x & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & x & \overline{x} & \overline{x} & \overline{x} & x & x & x & \overline{x} & \overline{x} & x & \overline{x} & \overline{e} & \overline{x} & x & \overline{x} & a & b & c & d & x & \overline{x} & x & x & \overline{x} & \overline{x} & \overline{x} & x \\
x & \overline{x} & x & x & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & x & \overline{x} & \overline{x} & x & x & x & \overline{x} & x & x & \overline{x} & x & \overline{x} & x & x & x & \overline{x} & e & x & x & \overline{b} & a & \overline{d} & c & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & x & \overline{x} & \overline{x} \\
x & \overline{x} & \overline{x} & \overline{x} & x & x & \overline{x} & x & \overline{x} & x & x & x & \overline{x} & \overline{x} & x & \overline{x} & x & x & x & \overline{x} & x & \overline{x} & x & x & x & \overline{x} & e & x & \overline{c} & d & a & \overline{b} & x & x & \overline{x} & x & \overline{x} & x & x & x \\
x & x & \overline{x} & x & \overline{x} & x & x & x & \overline{x} & \overline{x} & x & \overline{x} & x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & \overline{x} & x & x & x & \overline{x} & x & x & x & \overline{e} & \overline{d} & \overline{c} & b & a & \overline{x} & x & x & x & \overline{x} & \overline{x} & x & \overline{x} \\
x & x & x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & x & x & x & \overline{x} & x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & x & x & \overline{x} & \overline{x} & x & \overline{x} & x & a & b & c & \overline{e} & d & x & x & x \\
x & \overline{x} & x & x & \overline{x} & x & \overline{x} & \overline{x} & x & \overline{x} & x & x & \overline{x} & \overline{x} & x & \overline{x} & \overline{x} & x & x & x & \overline{x} & x & \overline{x} & \overline{x} & x & x & x & x & x & x & \overline{x} & \overline{x} & \overline{b} & a & e & c & x & \overline{d} & \overline{x} & x \\
x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & x & x & x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & x & \overline{x} & x & x & \overline{x} & x & x & x & x & \overline{x} & x & \overline{x} & \overline{x} & x & x & \overline{x} & \overline{c} & \overline{e} & a & \overline{b} & x & x & \overline{d} & \overline{x} \\
x & x & \overline{x} & x & \overline{x} & \overline{x} & x & \overline{x} & x & x & \overline{x} & x & x & \overline{x} & x & x & x & x & x & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & x & x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & \overline{x} & e & \overline{c} & b & a & \overline{x} & x & \overline{x} & d \\
\overline{x} & x & x & x & \overline{x} & x & x & x & x & x & x & \overline{x} & \overline{x} & x & x & x & \overline{x} & x & x & x & x & \overline{x} & x & x & x & x & \overline{x} & \overline{x} & x & x & x & x & \overline{d} & \overline{x} & \overline{x} & x & a & b & c & e \\
x & x & \overline{x} & x & x & x & \overline{x} & x & x & \overline{x} & x & x & x & x & \overline{x} & x & x & x & \overline{x} & x & \overline{x} & \overline{x} & \overline{x} & x & x & \overline{x} & x & \overline{x} & x & \overline{x} & \overline{x} & x & \overline{x} & d & \overline{x} & \overline{x} & \overline{b} & a & \overline{e} & c \\
x & x & x & \overline{x} & x & x & x & \overline{x} & x & \overline{x} & \overline{x} & \overline{x} & x & x & x & \overline{x} & x & x & x & \overline{x} & x & x & \overline{x} & x & \overline{x} & \overline{x} & \overline{x} & \overline{x} & x & x & \overline{x} & \overline{x} & \overline{x} & x & d & x & \overline{c} & e & a & \overline{b} \\
\overline{x} & x & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & \overline{x} & x & x & \overline{x} & x & \overline{x} & x & \overline{x} & \overline{x} & \overline{x} & x & \overline{x} & \overline{x} & \overline{x} & x & x & x & x & \overline{x} & \overline{x} & x & \overline{x} & x & \overline{x} & x & \overline{x} & \overline{x} & x & \overline{d} & \overline{e} & \overline{c} & b & a
\end{bmatrix}
$$

An $OD(40; 1, 1, 1, 1, 1, 35)$

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