Program
Alberta-Montana Combinatorics and Algorithms Days
Banff International Research Station, Banff, Alberta

Friday June 3	**Speaker or Event**	**Title**
by evening	*Check-in & Registration at the front desk in the Professional Development Centre*	
evening	*Informal gathering*	

Saturday June 4	**Speaker or Event**	**Title**
8:25-8:30	*Welcome address by BIRS staff*	
8:30-9:00	Ting Han Wei, University of Alberta	From Playing to Solving Go
9:00-9:30	Thomas Pender, University of Lethbridge	Balancedly Splittable Orthogonal Designs
9:30-10:00	Anastasia Halfpap, University of Montana	Positive co-degree and unusual stability
10:00-10:30	*Coffee break*	
10:30-11:30	Mark Kayll, University of Montana	KE and Egerváry graphs: a stability structure graph decomposition
11:30-12:00	Kris Vasudevan, University of Calgary	Success and challenges with the use of graph theory in brain disorder studies: Seeking answers
12:00-1:15pm	*Lunch*	
1:15-1:30	*Group Photo in TCPL Foyer near the talk venue (or outside weather permitting)*	
1:30-2:30	Joy Morris, University of Lethbridge	Cop Numbers of Generalised Petersen Graphs
2:30-3:00	Cory Palmer, University of Montana	At most 3.55^n stable matchings
3:00-3:30	*Coffee Break*	
3:30-4:00	Bobby Miraftab, University of Lethbridge	Median-decompositions and their applications
4:00-5:00	Micheal Cavers, University of Calgary	Reconfiguring vertex colourings of graphs
5:00-5:30	Vlad Zaitsev, University of Lethbridge	A class of optimal constant weight ternary codes
5:30-6:00	Ramin Mousavi, University of Alberta	Some Advances in the Planar Directed Steiner Tree Problem
6:00-7:30	*Dinner*	
7:30-8:30	Mathematics Education in Canada, presented by Rob Craigen and followed by discussion	

Sunday June 5	**Speaker or Event**	**Title**
8:30-9:00	Rob Craigen, University of Manitoba	Critical cases of circulant partial Hadamard matrices
9:00-10:00	Ryan Hayward, University of Alberta	Hex problems
10:00-10:30	*Coffee break (note: room check out is by 11:00)*	
10:30-11:30	Zachary Friggstad, University of Alberta	Prize-Collecting Walks and Branchings in Directed Graphs
11:30-12:00	Davoud Abdi, University of Calgary	Siblings of Countable *NE*-free Posets
12:05pm	*Lunch*	

ABSTRACTS

Alberta-Montana Combinatorics and Algorithms Days
Banff International Research Station
Banff, Alberta
Friday June 3 – Sunday June 5, 2022

Invited Speakers (in alphabetical order by speaker's name)

Reconfiguring vertex colourings of graphs

Michael Cavers
Department of Mathematics and Statistics
University of Calgary

Given two feasible solutions to a problem, a *reconfiguration problem* asks if one solution can be transformed to the other through a sequence of allowable steps, while maintaining feasibility at each step. Reconfiguration problems have recently received wide attention, and have been studied for various graph problems such as colouring, dominating sets, vertex covers, cliques and independent sets.

The focus on this talk is on the reconfiguration of k-colourings of graphs. To be precise, given a graph $H = (V,E)$ and a positive integer k, a *k-colouring* of H is a function $f : V(H) \to \{1,2,\ldots,k\}$ for which $f(x) \neq f(y)$ for any $xy \in E(H)$. The *k-colouring graph of H*, denoted by $G_k(H)$, is the graph whose vertices correspond to all k-colourings of H, and whose edges connect two k-colourings of H that differ on exactly one vertex of H. The k-colouring graph is an example of a *reconfiguration graph* and it arises in the context of theoretical physics (it is the graph of the Glauber dynamics markov chain where the goal is to find efficient algorithms for almost uniform sampling of k-colourings of graphs). We highlight recent results and conclude with the problem of determining when $G_k(H)$ has a Hamilton cycle.

Prize-Collecting Walks and Branchings in Directed Graphs

Zac Friggstad
Department of Computing Science
University of Alberta

Consider a directed graph having edge costs and node penalties. The *prize-collecting* cost of a walk is the cost of its edges plus the penalty of unvisited nodes. While it is **NP**-hard to find the cheapest prize-collecting walk, an arborescence-packing result by Bang-Jensen, Frank, and Jackson (1995) is readily adapted to show the following: For any node r, in polynomial time we can find an r-branching (directed tree pointing away from r) whose prize-collecting cost is at most the cheapest prize-collecting walk starting at r. This has recently found applications in the design of approximation algorithms for vehicle routing problems. However, these algorithms involve solving linear programs (LPs) with a prohibitive number of variables and constraints.

In this talk, I will first discuss the packing result and its applications to algorithm design. Then a new algorithm for finding branchings with low prize-collecting cost will be presented. The algorithm is efficient in practice, purely combinatorial, and can be used instead of an LP solver in some vehicle routing algorithms while still maintaining the approximation guarantee of the algorithm. This is joint work with Sina Dezfuli, Ian Post, and Chaitanya Swamy.

Hex problems

Ryan Hayward
Department of Computing Science
University of Alberta

Who invented Hex? When did Martin Gardner tell his readers about Hex? Did Gardner believe that John Nash had independently re-invented Hex? What roles do Karen Thorborg and Jens Lindhard play in this story? We will overview *Hex, the Full Story* (CRC Press 2019) and *Hex, a Playful Introduction* (MAA Press 2022) and mention some open Hex problems. Time permitting, we will also mention some Go and Clobber problems.

KE and Egerváry graphs: a stability structure graph decomposition

P. Mark Kayll

Department of Mathematical Sciences
University of Montana

Kőnig-Egerváry (KE) graphs—which generalize bipartite graphs—have matching (ν) and stability (α) numbers summing to their order. Deming (1979) developed an efficient algorithm to recognize these graphs and to find a maximum stable set in a KE graph. *Egerváry* graphs (aka 'BvN graphs') are those for which the blossom constraints are implied by the other constraints in Edmonds' (1965) perfect matching polytope theorem. These graph classes exhibit nice interconnections. For example, the Birkhoff-von Neumann theorem on doubly-stochastic matrices is equivalent to bipartite graphs being Egerváry. More generally, KE graphs are Egerváry. This talk introduces these ideas and shares a few related results. In particular, we discuss our extension of Deming's algorithm to produce a decomposition of a matchable input graph into matchable subgraphs which are 'almost KE' (with $\alpha = \nu - 1$), together with a residual matchable KE subgraph (with $\alpha = \nu$). (Joint work with JACK EDMONDS and CRAIG LARSON.)

Cop Numbers of Generalised Petersen Graphs

Joy Morris

Department of Mathematics and Computer Science
University of Lethbridge

Cops and Robbers is a two-player pursuit/evasion game that can be played on any graph, with the graph serving as the network that determines the permissible moves from any location. Cops alternate turns with the robber, with all cops potentially moving on their turn. Both sides have perfect information, and the cops' goal is for one of them to land on the robber, while the robber's goal is to evade capture forever. The cop number of a graph is the minimum number of cops required in order to ensure that a winning strategy exists for the cops.

It was previously proved by Ball et al. (2015) that the cop number of any generalised Petersen graph is at most 4. I will present results that explain all of the known generalised Petersen graphs that actually have cop number 4, and that place them in the context of infinite families. A key consideration is the girth of the graph. This is based on joint work with Harmony Morris, Tigana Runte, and Adrian Skelton.

Contributed Talks (in alphabetical order by speaker's name)

Siblings of Countable NE-free Posets

Davoud Abdi
Department of Mathematics and Statistics
University of Calgary

Two structures R and S are *equimorphic* when each embeds in the other; we may also say that one is a *sibling* of the other. Generally, it is not the case that equimorphic structures are necessarily isomorphic: the rational numbers, considered as a linear order, has up to isomorphism continuum many siblings. Let $Sib(R)$ be the number of siblings of R, these siblings are counted up to isomorphism. Thomassé conjectured that for each countable relational structure R, made of at most countably many relations, $Sib(R) = 1$, $\aleph_0$ or $2^{\aleph_0}$. There is an alternative case of interest, namely whether $Sib(R) = 1$ or infinite for a relational structure R of any cardinality.

In this talk, I will mention those structures for which the conjectures have been partially or completely verified. Then, I will intoduce NE-free posets, classify them and give a sketch of proof of the alternative Thomassé's conjecture for countable NE-free posets.

Critical cases of circulant partial Hadamard matrices

R. Craigen
Department of Mathematics
University of Manitoba

A *circulant partial Hadamard matrix* is a (rectangular) circulant matrix $H \in \{\pm 1\}^{k\times n}$ satisfying

$$HH^\top = nI_k.$$

A third parameter r gives the sum along the first row of H, from which all rows are generated. We use r-$H(k \times n)$ to denote such a matrix.

For which h,k,r does r-$H(k \times n)$ exist? Since r-$H(k \times n)$ implies r-$H(h \times n)$ for all $1 \leq h < k$, it suffices to determine the maximum value of k, given r and n.

The pattern of maximum k values over the full range of r and k suggests, among other things, a possible resolution of the circulant Hadamard matrix conjecture.

Several relations between the three parameters may be derived, the chief of which is $r\sqrt{k} \leq n$. When this bound is approached from various directions, remarkable behaviours emerge at certain threshold values, forcing greatly simplified structure. Some cases prove to be equivalent to other known configurations of interest. Critical cases appear to govern the overall pattern of existence of these matrices.

We show the derivation of some new critical cases and the intriguing pattern they occupy in the larger landscape.

Positive co-degree and unusual stability

Anastasia Halfpap

Department of Mathematical Sciences
University of Montana

Let F be a fixed graph and let $ex(n,F)$ be the maximum number of edges in an n-vertex, F-free graph G. In many situations, in addition to well understanding $ex(n,F)$, we can prove stability results: statements of the form that if G is an n-vertex, F-free graph with

$$e(G) > ex(n,F) - f(n)$$

for some "small" function of n, then the edit distance between G and the n-vertex extremal graph for F must be correspondingly small.

The extension of stability questions to extremal hypergraph theory is very natural. However, one fundamental barrier in extremal hypergraph theory is that, far more often than in extremal graph theory, multiple very different extremal or near-extremal constructions may exist. In such situations, it may be assumed that no form of stability result is possible, but this is not always true.

In this talk, we introduce the problem of determining

$$co^{+}ex(n,F),$$

the largest possible *minimum positive co-degree* in an r-regular hypergraph free of a fixed r-graph F, highlighting the example of $F = K_4^-$, for which two extremal constructions exist. We then indicate a proof that, if H is a 3-graph whose minimum positive co-degree is $co^{+}ex(n,K_4^-) - o(n)$, then H must have edit distance $o(n^3)$ from one of the two extremal constructions. Joint work with Cory Palmer and Nathan Lemons, and with Ramon Garcia.

Median-decompositions and their applications

Bobby Miraftab

Department of Mathematics
University of Lethbridge

Median-decompositions have been introduced by Stavropoulos, as a generalization of tree-decompositions, where instead of decomposing a graph in a treelike fashion, general median graphs can be used as the underlying graph of the decomposition. His method for constructing of median decompositions is based on a Cartesian product of tree-decompositions. In this talk, we will show that there is always a way to construct a median-decomposition from an arbitrary set of separations which is more optimal. In addition, we will discuss the connection of median-decompositions with chordality and hyperbolicity. This is joint work with Aristotelis Chaniotis and Sophie Spirkl.

Some Advances in the Planar Directed Steiner Tree Problem

Ramin Mousavi

Department of Computing Science
University of Alberta

In the directed Steiner tree (DST) problem, we are given a directed graph with edge costs, a set of terminal nodes, and a root node r. The goal is to find a minimum-cost subgraph in which every terminal is reachable from the root. DST is a common generalization of many important problems in combinatorial optimization including set cover and facility location. But this generalization comes at a cost; optimal DST solutions are notoriously hard to even approximate.

In this talk, I discuss advances in an important special case of DST when the underlying graph is planar or, more generally, excludes a fixed minor. We consider the primal-dual scheme in which linear programming duality is leveraged to design approximately-optimal solutions. This scheme has been successfully applied to numerous combinatorial problems including the undirected version of DST. I talk about the challenges in applying the standard primal-dual algorithm for DST and how we can bypass these challenges to obtain the first constant-factor approximation algorithm for a certain special case of planar DST. This is joint work with Zachary Friggstad.

At most 3.55^n stable matchings

Cory Palmer

Department of Mathematical Sciences
University of Montana

In this talk we will discuss an improvement of 131072^n to 3.55^n on the upper bound on the maximum possible number of stable matchings among n jobs and n applicants. To establish new bounds we state a novel formulation of a certain entropy bound that is easy to apply and may be of independent interest in counting other combinatorial objects. Joint work with Dömötör Pálvölgyi.

Balancedly Splittable Orthogonal Designs

Thomas Pender

Department of Mathematics and Computer Science
University of Lethbridge

A Hadamard matrix is a square $(-1,1)$-matrix whose rows are pairwise orthogonal. Kharaghani and Suda (2019) termed such a matrix balancedly splittable if upon forming a submatrix from a subset of its rows, the columns of the new matrix form a set of vectors which are at most biangular. This idea is extended to apply to orthogonal designs among other generalizations of Hadamard matrices. Time permitting, we will explore connections to related objects. This is a joint work with Hadi Kharaghani and Sho Suda.

From Playing to Solving Go

Ting Han Wei
Department of Computing Science
University of Alberta

Since AlphaGo's winning match against human champion Lee Sedol in 2016, computer Go programs that play at super-human levels have now become widely available. Playing and solving Go are two separate yet closely related tasks. Playing usually involves making decisions that maximize one's probability of winning, while solving requires knowledge of exact outcomes, regardless of opponent strategy. For playing Go, since AlphaGo's achievements in 2016, numerous super-human level programs can now be obtained, or even trained from scratch with just a few dozen GPUs over the course of several months. In contrast, solving the game is a much more difficult problem. To illustrate the enormity of the task, the number of legal moves in a standard game of 19x19 Go is about 2×10^{170}. Indeed, the latest results from 2009 solved boards of only up to 5x6.

In this talk, I will present some obstacles and methods that can be used to potentially solve 6x6 Go using search-based methods. Details will include dealing with the so-called graph history interaction problem (GHI), using static safety to reduce the search space, designing or learning heuristics to guide the search, and the decomposition search technique.

Success and challenges with the use of graph theory in brain disorder studies: Seeking answers

Kris Vasudevan

Department of Mathematics and Statistics
University of Calgary

Graph theory analysis is successfully used to study functional connectivity between different regions of the brain with data from different experiments such as functional magnetic resonance imaging (fMRI), scalp and intracranial electroencephalogram (EEG and iEEG), and magneto encephalography (MEG). In this presentation, I restrict myself to the iEEG data obtained from subdural grid and depth electrodes implanted in epilepsy patients undergoing pre-surgical evaluation. Multiple sensors used in such studies are known to register neural activity from a single source, known as volume conduction. As a result, graph theoretic measures revolving around correlation and coherency, in spite of their success, present challenges to analysis and interpretation due to spatial leakage. Alternative measures such as phase lag index, imaginary coherency and corrected amplitude envelope correlation or dimensionality reduction approaches appear to deal with the spatial leakage issue. In my presentation, I examine the pros and cons of the use of traditional and new methods and metrics and their statistical significance with one particular class of seizures namely focal to bilateral tonic-clonic seizures.

This is joint work with Elena Braverman and Michael Cavers.

A class of optimal constant weight ternary codes

Vlad Zaitsev

Department of Mathematics and Computer Science
University of Lethbridge

A matrix W with entries in $\{0,1,-1\}$ for which $WW^t = pI_n$ is a weighing matrix of order n and weight p. A weighing matrix W of order $n = \frac{p^{m+1}-1}{p-1}$ and weight p^m is constructed and it is shown that the rows of W and $-W$ together form optimal constant weight ternary codes of length n, weight p^m and minimum distance $p^{m-1}(\frac{p+3}{2})$ for each odd prime power p and integer $m \geq 1$.

Joint work with Hadi Kharaghani and Sho Suda.