HAMILTONIAN PATHS IN CARTESIAN POWERS OF DIRECTED CYCLES

DAVE WITTE

ABSTRACT. The vertex set of the k^{th} cartesian power of a directed cycle of length m can be naturally identified with the abelian group $(\mathbb{Z}_m)^k$. For any two elements $u = (u_1, \ldots, u_k)$ and $v = (v_1, \ldots, v_k)$ of $(\mathbb{Z}_m)^k$, it is easy to see that if there is a hamiltonian path from u to v, then

$$u_1 + \cdots + u_k \equiv v_1 + \cdots + v_k + 1 \pmod{m}.$$

We prove the converse, unless $k = 2$ and m is odd. This is joint work with David Austin and Heather Gavlas. A similar result is conjectured for cartesian products of directed cycles that are not assumed to be of equal lengths.