

# VERTEX EMBEDDINGS OF REGULAR POLYTOPES

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ABSTRACT. Starting from the fact, known since antiquity, that it is possible to choose four vertices of the cube so as to form the vertices of a regular tetrahedron, we investigate in this mainly expository talk the general question of when the vertices of one regular polytope embed in those of another regular polytope. Relationships of this question with several areas of mathematics will be discussed, including combinatorics, linear algebra, number theory, Galois theory, and algebraic topology. The proof of the equivalence of the following three statements will be outlined:

- (1) the regular  $(n - 1)$ -simplex has a vertex embedding in the  $(n - 1)$ -cube,
- (2) there exists a Hadamard matrix of order  $n$ ,
- (3) the regular  $n$ -orthoplex (generalized octahedron) has a vertex embedding in the  $n$ -cube.