ABSTRACTS

Fourth Combinatorics Day
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Graph Coloring Got Married

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This will be an expository/survey talk about a recent, very interesting generalization of graph coloring. Some of you will surely be able to conclude from the title what I am referring to. For those of you who can't, I won't give it away, and if I have aroused your interest, you will just have to come to the talk to find out.

Boolean filtering of ternary structures

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Many combinatorial structures elude the formation of complete systematic theories about their structure, perhaps because they are being examined at the wrong level of generality, or unsuitable for the tools at hand, or perhaps just because they are inherently difficult to pin down. When we have need to understand such a structure, let us say an object that must satisfy some set of combinatorial conditions, the following three problems automatically arise: When do they exist? How many are there? Classify them.

One's first reaction might be that, since a complete answer to each of the latter two questions implies an answer to the former, the first question is the easiest and the third is hardest. So if a theory is difficult to obtain, one should concentrate on the easy question. This is often a mistake, because the three questions, especially the first and third, cannot easily be separated from each other, and often none of them can be solved without at least a partial answer to the others.

The following two strategies may be useful in addressing these issues when faced with a difficult problem of this type:

1. Add more conditions. This results in objects with much stronger properties, and structure, than the object being studied. It may be that this stronger structure is easier to study, and may yield at least a solution to an important special case. If one chooses the extra conditions carefully, one may even obtain a complete solution to the existence question. In any case, when the stronger conditions can be satisfied, the resulting objects are considered "better" than the general case, and this is certainly a consideration from the perspective of applications.

2. Take away, or weaken, some conditions. This strategy is followed less often, possibly because it is tempting to believe that this makes the problem harder because it increases the size of the class of objects under study. The benefit to this approach is that a well-chosen generalization or weakening of the structure will lead to a coarser understanding of the objects, enable us to see the "forest" rather than the "trees". In any case, generalizations or partial structures can give us an "outer envelope" for structures in which we are interested, when the exact form of their theories is hard to obtain. General questions are easier to answer than specific ones about as often as the other way around, and a further consequence of generality is that any facts obtained apply to more situations than that from which they arise.

In this talk I wish to focus on two types of objects, and a strategy of the latter type above for studying them. We consider weighing matrices, which are square $(0,1,-1)$-matrices A such that $AA^t = wI$ for some integer w. The other structure is ternary complementary pairs, which are pairs of $(0,1,-1)$-sequences whose (joint) autocorrelation is zero. Both structures have well-studied partial theories, and both have eluded complete analysis. I will summarize some recent work in which we simplify both structures by merely concentrating on where the 0's go, by the simple device of working modulo 2. Thus, we examine square $(0,1)$-matrices and $(0,1)$-sequences with some simple algebraic structure, and obtain coarse results for the ternary case. Since the resulting objects can be defined in terms of simple boolean algebraic conditions we fondly refer to this technique as "boolean filtering", anticipating that the same technique may be useful in other settings where ternary structures appear.

Surprisingly, the objects resulting from our boolean filtering are interesting and elegant, and probably worthy of study on their own.

Generalized Conference Matrices

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A generalized conference matrix $GC(G;n)$ over a finite (multiplicatively written) group G of order g is a matrix $C = [c_{ij}]$ of order $ng+2$ with $c_{ii} = 0$ and $c_{ij} \in G$ for $i \neq j$ such that, for any distinct i and h, the multiset $\{c_{ij}c_{hj}^{-1} : j \neq i, j \neq h\}$ contains n copies of every element of G. We will consider examples of generalized conference matrices over abelian and non-abelian groups and their relations with other combinatorial designs and pose some questions.

Hamiltonian cycles in circulant graphs and digraphs

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We will discuss the status of the search for hamiltonian cycles in circulant graphs and circulant digraphs. Circulant graphs have many hamiltonian cycles, but recent joint work with Joy Morris and David Moulton uncovered a nontrivial parity condition that restricts the hamiltonian cycles in certain cases. A different parity condition arose in joint work with Stephen Locke that constructed infinitely many circulant digraphs with no hamiltonian cycles. The case of digraphs remains largely open.