Let $f\colon \mathbf{R}^3 \to \mathbf{R}$ and $\mathbf{F}\colon \mathbf{R}^3 \to \mathbf{R}^3$. Write $\mathbf{F} = (f_1, f_2, f_3)$.
Similarly for g and $\mathbf{G}$.
Define:

$$\begin{aligned}
\operatorname{grad}(f) \equiv \nabla f &= \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) \\
\operatorname{div}(\mathbf{F}) \equiv \nabla \cdot \mathbf{F} &= \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z} \\
\operatorname{curl}(\mathbf{F}) \equiv \nabla \times \mathbf{F} &= \left(\frac{\partial f_3}{\partial y} - \frac{\partial f_2}{\partial z}, \frac{\partial f_1}{\partial z} - \frac{\partial f_3}{\partial x}, \frac{\partial f_2}{\partial x} - \frac{\partial f_1}{\partial y}\right) \\
\operatorname{laplace}(f) \equiv \nabla^2 f &= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \\
\operatorname{laplace}(\mathbf{F}) \equiv \nabla^2 \mathbf{F} &= \left(\nabla^2 f_1, \nabla^2 f_2, \nabla^2 f_3\right)
\end{aligned}$$

The following properties hold:

$$\begin{aligned}
\operatorname{grad}(f+g) \equiv \nabla(f+g) &= \nabla f + \nabla g \\
\operatorname{div}(\mathbf{F}+\mathbf{G}) \equiv \nabla \cdot (\mathbf{F}+\mathbf{G}) &= \nabla \cdot \mathbf{F} + \nabla \cdot \mathbf{G} \\
\operatorname{curl}(\mathbf{F}+\mathbf{G}) \equiv \nabla \times (\mathbf{F}+\mathbf{G}) &= \nabla \times \mathbf{F} + \nabla \times \mathbf{G} \\
\operatorname{grad}(fg) \equiv \nabla(fg) &= f\nabla g + g\nabla f \\
\operatorname{div}(f\mathbf{G}) \equiv \nabla \cdot (f\mathbf{G}) &= \nabla f \cdot \mathbf{G} + f\nabla \cdot \mathbf{G} \\
\operatorname{curl}(f\mathbf{G}) \equiv \nabla \times (f\mathbf{G}) &= \nabla f \times \mathbf{G} + f\nabla \times \mathbf{G} \\
\operatorname{grad}(\mathbf{F}\cdot\mathbf{G}) \equiv \nabla(\mathbf{F}\cdot\mathbf{G}) &= (\mathbf{F}\cdot\nabla)\mathbf{G} + (\mathbf{G}\cdot\nabla)\mathbf{F} + \mathbf{F}\times(\nabla\times\mathbf{G}) + \mathbf{G}\times(\nabla\times\mathbf{F}) \\
\operatorname{div}(\mathbf{F}\times\mathbf{G}) \equiv \nabla \cdot (\mathbf{F}\times\mathbf{G}) &= \mathbf{G}\cdot\nabla\times\mathbf{F} - \mathbf{F}\cdot\nabla\times\mathbf{G} \\
\operatorname{curl}(\mathbf{F}\times\mathbf{G}) \equiv \nabla \times (\mathbf{F}\times\mathbf{G}) &= \mathbf{F}(\nabla\cdot\mathbf{G}) - \mathbf{G}(\nabla\cdot\mathbf{F}) + (\mathbf{G}\cdot\nabla)\mathbf{F} - (\mathbf{F}\cdot\nabla)\mathbf{G} \\
\operatorname{div}\operatorname{grad} f \equiv \nabla\cdot\nabla f &= \nabla^2 f = \operatorname{laplace} f \\
\operatorname{curl}\operatorname{grad} f \equiv \nabla\times\nabla f &= 0 \\
\operatorname{div}\operatorname{curl}\mathbf{F} \equiv \nabla\cdot(\nabla\times\mathbf{F}) &= 0 \\
\operatorname{curl}^2\mathbf{F} \equiv \nabla\times(\nabla\times\mathbf{F}) &= \nabla\nabla\cdot\mathbf{F} - \nabla^2\mathbf{F} = \operatorname{grad}\operatorname{div}\mathbf{F} - \operatorname{laplace}\mathbf{F} \\
\operatorname{grad}\operatorname{div}\mathbf{F} \equiv \nabla\nabla\cdot\mathbf{F} &= \nabla\times(\nabla\times\mathbf{F}) + \nabla^2\mathbf{F} = \operatorname{curl}^2\mathbf{F} + \operatorname{laplace}\mathbf{F}
\end{aligned}$$

The other combinations, grad^2, div^2 and $\operatorname{grad}\operatorname{curl}$ are meaningless.