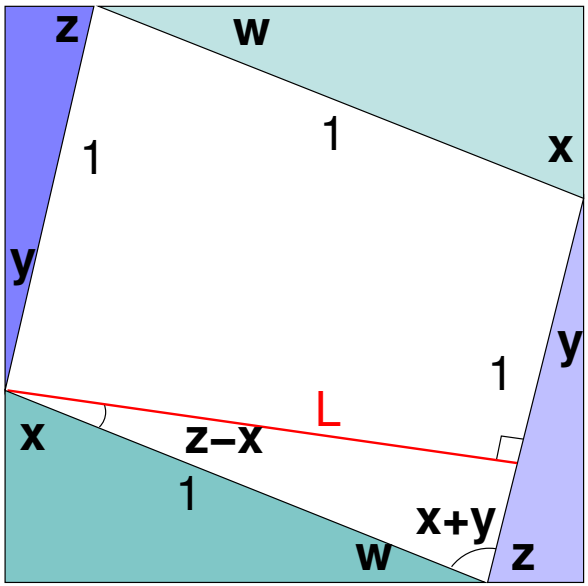
Proof of the Sine and Cosine Addition Formulas

References: Priebe & Ramos, Math Mag 73:5 (2000) pg 392
Webber & Bode, Math Mag 75:5 (2002) pg 398

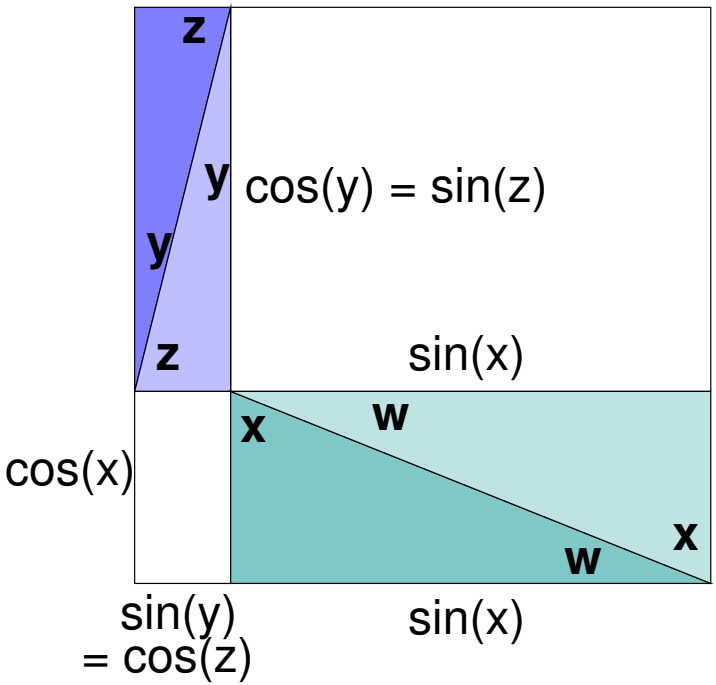
Consider two (green) right triangles with hypotenuse 1 and angle x,
and two (blue) triangles with hypotenuse 1 and angle y.

Arrange them in two ways below to form squares of the same area.

The indicated angle is x+y since w + (x+y) + z = (w+x) + (y+z) = 90 + 90 = 180 degrees.
The other angle is z−x, because (z−x) + (x+y) = y+z = 90 degrees.



The white area is 1*L = sin(x+y) = cos(z−x)



The two white areas sum to

cos(x)sin(y)+sin(x)cos(y) or
cos(x)cos(z)+sin(x)sin(z)