Number Theory Day
University of Lethbridge - May 07th, 2008
Abstracts

Hugh Williams - *A cubic extension of the Lucas functions.*
From 1876 to 1878 Lucas developed his theory of the functions V_n and U_n, which now bear his name. He was particularly interested in how these functions could be employed in proving the primality of certain large integers, and as part of his investigations succeeded in demonstrating that the Mersenne number $2^{127} - 1$ is a prime. V_n and U_n can be expressed in terms of the nth powers of the zeros of a quadratic polynomial, and throughout his writings Lucas speculated about the possible extension of these functions to those which could be expressed in terms of the nth powers of the zeros of a cubic polynomial. Indeed, at the end of his life he stated that "by searching for the addition formulas of the numerical functions which originate from recurrence sequences of the third or fourth degree, and by studying in a general way the laws of the residues of these functions for prime moduli we would arrive at important new properties of prime numbers." We have very little idea of what functions Lucas had in mind because he provided so little information concerning this in his published and unpublished work.
In this talk we will discuss a pair of functions that are easily expressed in terms of the zeros of a cubic function and show how they do seem to provide an extension of Lucas' theory. We will do this by developing analogs involving these new functions for all the principal results of Lucas' theory concerning V_n and U_n.
This is joint work with Siguna Müller (University of Wyoming) and Eric Roettger (University of Calgary).

Matilde Lalin - *Higher Mahler measures.*
The classical Mahler measure of an n-variable polynomial P is the integral of $\log|P|$ over the n-dimensional unit torus T^n with the Haar measure. We consider, more generally, the integral of $\log^k|P|$. Specific examples yield special values of zeta functions, Dirichlet L-series, and polylogarithms.
This is a joint work with N. Kurokawa and H. Ochiai.

Greg Martin - *Prime number races.*
This talk is a survey of "prime number races". Chebyshev noticed in the first half of the nineteenth century that for any given value of x, there always seem to be more primes of the form $4n+3$ less than x then there are of the form $4n+1$. Similar observations have been made with primes of the form $3n+2$ and $3n+1$, primes of the form $10n+3/10n+7$ and $10n+1/10n+9$, and many others besides. More generally, one can consider primes of the form $qn+a$, $qn+b$, $qn+c$, ... for our favorite constants q, a, b, c, ... and try to figure out which forms are "preferred" over the others - not to mention what, precisely, being "preferred" means. We describe these phenomena in greater detail and explain the efforts that have been made at understanding them.

Dragos Ghioca - *A dynamical problem in arithmetic geometry.*
We present a dynamical version of the Mordell-Lang conjecture for subvarieties of quasiprojective varieties, endowed with the action of an endomorphism.

Brandon Fodden - *Diophantine equations and the generalized Riemann hypothesis.*
We indicate how one may show that the statement "the generalized Riemann hypothesis holds for all number fields K" is equivalent to the unsolvability of a particular Diophantine equation. This is done using methods arising from the negative solution to Hilbert's tenth problem. These methods allow one to show that statements of a certain type are equivalent to the unsolvability of a Diophantine equation.