Problem Solving - October 31 - 3:00-4:50 - B650

Hallowe'en

Guidelines

Eat Candy. Have fun. There are a lot of problems. Don't try to do all of them. Don't do any you already know how to solve. Work in groups. Don't give up after five minutes. Plug in small numbers. Look for patterns. Draw pictures. Use symmetry. Try cases. Work backwards. If you're stuck, take a break and play a game. Find equivalent versions of the problem. Choose effective notation.

Problems

1. Gargamel has captured 3 smurfs but will allow them to play the following game for their freedom: He removes their hats and replaces them with hats that are either green or purple. The probability that the hat is green is equal to the probability that the hat is purple, and both are equal to one half. In order to win their freedom the smurfs must obey the following rules:

   - They may not communicate with each other once the game has started. They may develop a strategy beforehand, though.

   - Each smurf can see the hat colours of all the other smurfs but not their own and they cannot see any gestures, hear the others, or otherwise communicate with each other. They can only see the colour of the hat.

   - In order to win the game, at least one smurf must guess the colour of their hat (the others may pass), and no smurf can guess incorrectly.

   What is an optimal strategy for the smurfs?

2. It's Hallowe'en, and you have N pieces of the same candy to give $k \leq N$ kids who come to the door wanting candy all with the same costume. How many ways can you distribute all the candy among the k kids? How many can you distribute ways such that every receives at least one piece of candy?

3. A Velociraptor spots Adam 40 meters away and attacks immediately accelerating at 4 m/s^2 to a top speed of 25 m/s. At the same time Adam instantly bikes away at 10 m/s. Assuming a near instantaneous death once the velociraptor reaches Adam, how long before he dies an excruciating death?

4. Adam, now a zombie (who knew that velociraptors were zombies?), is being chased by the zombies three natural enemies: 1 unicorn and 2 rednecks. They have surrounded him at the centre of an equilateral triangle with sides equal to 20 m. The two rednecks and unicorn accelerate at 3 m/s^2 to their maximum speed: 10 m/s for the unicorn and 25 for the rednecks driving their pickup trucks. Assuming Adam starts biking (yes, he still has his bike) at a constant speed of 6 m/s, at what angle should he bike to maximize his survival time?

---

5. (Putnam 1989 B1) A dart, thrown at random, hits a square target. Assuming that any two parts of the target of equal area are equally likely to be hit, find the probability that the point hit is nearer to the centre than to any edge. Express your answer in the form $(a\sqrt{b}+c)/d$, where a, b, c, d are positive integers.

6. (Putnam 1991 A1) A 2×3 rectangle has vertices at $(0,0), (2,0), (0,3)$, and $(2,3)$. It rotates $90°$ clockwise about the point $(2,0)$. It then rotates $90°$ clockwise about the point $(5,0)$, then $90°$ clockwise about the point $(7,0)$, and finally $90°$ clockwise about the point $(10,0)$. (The side originally on the x-axis is now back on the x-axis.) Find the area of the region above the x-axis and below the curve traced out by the point whose initial position is $(1,1)$.