Problem Solving Session

Friday, November 22, 2013

3:00pm-4:50pm in B650

Here are this week's polynomial problems:

1. In the expansion of $\left(x^3 - 2x^2 - x + 2\right)^4$, what is...
   (a) ...the constant term?
   (b) ...the sum of the coefficients and the constant term?
   (c) ...the coefficient of x?

2. Find all ordered pairs of real numbers (x, y) that satisfy both $x^3 y = -4$ and $2x + x^2 y = 2$.

3. Find two real numbers x and y such that $x < y$, $x + y = 4$, and $x^3 + y^3 = 76$.

4. Find a polynomial with integer coefficients of which $\sqrt[3]{3} - \sqrt{2}$ is a root.

5. Find the number a with the property that $f(a) = a$ is a local minimum of
$$f(x) = x^4 - 4x^3 + 3x^2 + 2ax - 1.$$

6. A polynomial of degree n whose variable is x is called *monic* if the coefficient of x^n is 1. If $P(x)$ is a monic polynomial of degree 3 such that $P(1) = 1$, $P(2) = 4$, and $P(3) = 9$, what is $P(4)$?

7. A monic polynomial is called *peculiar* if its coefficients are in arithmetic progression and its roots are integers. One example is $x^2 - 1$, whose coefficients are 1, 0, -1 and whose roots are -1 and 1. Find all peculiar polynomials of degree 2.

8. Let a, b, c, d, and r be nonzero complex numbers such that r is a root of both
$$ax^3 + bx^2 + cx + d = 0 \quad \text{and} \quad bx^3 + cx^2 + dx + a = 0.$$
Find all possible values of r.

9. Let $P(x)$ be a polynomial with integer coefficients that is equal to 2010 for four distinct integral values of x. Show that there is no integer x for which $P(x)$ is equal to 2007.