Problem Solving Session

Friday, September 19, 2014

2:00pm-2:50pm in C630

The *AM-GM inequality* states that the arithmetic mean of nonnegative real numbers $x_1, x_2, \ldots x_n$ is greater than or equal to their geometric mean; that is,

$$\frac{x_1 + x_2 + \cdots + x_n}{n} \geq \sqrt[n]{x_1 x_2 \cdots x_n}.$$

Equality is achieved if and only if $x_1 = x_2 = \cdots = x_n$.

1. Prove the AM-GM inequality in the case of two variables i.e. $\dfrac{a+b}{2} \geq \sqrt{ab}$.

2. A farmer wants to fence in 60,000 square feet of land in a rectangular plot along a straight highway. The fence he plans to use along the highway costs \$2 per foot, while the fence for the other three sides costs \$1 per foot. How much of each type of fence will he have to buy in order to keep expenses to a minimum? What is the minimum expense?

3. Find the minimum value of $f(x) = \dfrac{144x^2 \sin^2 x + \pi^2}{x \sin x}$ on $(0, \pi)$.

4. Let the function f be positive and continuous on $\mathbb{R}$ such that $f(x+1) = f(x)$ for all real x. Prove that
$$\int_0^1 \frac{f(x)}{f(x+\frac{1}{2})}\, dx \geq 1.$$

5. Find the maximum value of $f(x) = (1-x)(1+x)(1+x)$ on $[0, 1]$.

6. Given a rectangle of width a and height b, use a straight edge and compass to construct a square of area ab.