Problem Solving Session

Friday, October 3, 2014

2:00pm-2:50pm in C630

The *Pigeonhole Principle* states that if m objects are placed in n boxes with $m > n$, then at least two objects will be placed in the same box.

1. Given any three integers, prove that the sum of two of them is even. Prove or disprove that the sum of two of them is odd.

2. Prove: if 34 distinct numbers are selected from $\{1, 2, 3, \ldots 99\}$, then two of the selected numbers will differ by at most 2.

3. Let S be a set of 10 positive integers not exceeding 50. Show that S has two different (but not necessarily disjoint) subsets of size 4 that have the same sum.

4. Prove: if 51 distinct numbers are selected from $\{1, 2, 3, \ldots 100\}$, then one of the selected numbers will divide another selected number.

5. Prove: if 27 distinct numbers are selected from $\{2, 3, 4, \ldots 100\}$, then two of the selected numbers will not be relatively prime.

6. I reach into a jar containing 13 pennies, 12 nickels, 9 dimes, and 8 quarters and blindly remove a bunch of coins. How many coins must I remove to guarantee that I end up with…

   (a) …3 pennies?

   (b) …5 coins of the same kind?

   (c) …2 nickels and 2 dimes?

   (d) …2 coins of one kind and 2 coins of another kind?

7. Given any 5 points on a sphere, show that at least 4 lie in the same closed hemisphere.