Problem Solving Session
Friday, October 10, 2014
2:00pm-2:50pm in C630

1. Find all the possible values of a/b where $a^2 - 6ab + 8b^2 = 0$ and $b \neq 0$.

2. Find all integer solutions to $m^2 = n^6 + 17$.

3. Let $P(x) = x^2 + bx + c$ be a polynomial satisfying $P(P(-1)) = P(P(2)) = 0$ and $P(-1) \neq P(2)$. Find $P(0)$.

4. Let $f(x)$ be a fourth degree monic polynomial such that $f(-1) = -1$, $f(2) = -4$, $f(3) = -9$, and $f(4) = -16$. Find $f(1)$.

5. How many positive solutions does $x^5 + 11x^4 + 17x^3 - 19x^2 = 0$ have? How many positive integer solutions does it have?

6. Find relatively prime positive integers r and s such that $\dfrac{r}{s} = \dfrac{2\left(\sqrt{2}+\sqrt{10}\right)}{5\left(\sqrt{3+\sqrt{5}}\right)}$.

7. Find the smallest positive integer n such that $x^4 + n^2$ is not prime for any integer x.

8. **(B1 2005)** Find a non-zero polynomial $P(x,y)$ such that $P(\lfloor t \rfloor, \lfloor 2t \rfloor) = 0$ for all real numbers t.

9. **(B1 2004)** Let $p(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$ be a polynomial with integer coefficients. If r is a rational root of $p(x)$, show that the numbers $a_n r$, $a_n r^2 + a_{n-1} r, \ldots, a_n r^n + a_{n-1} r^{n-1} + \ldots + a_1 r$ are all integers.