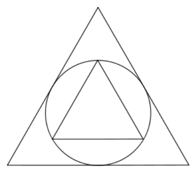
Problem Solving - October 17 - 2:00-2:50 - C630

Problems

Symmetry: Most of these problems can all be done with some kind of symmetry, but that is not the only way to do them. Some of the don't require any symmetry.

1. Inscribe a circle in an equilateral triangle and then inscribe an equilateral triangle in the circle. Using only a picture, find the ratio of the areas of the two triangles. Change the triangles into squares. What happens?



2. Add a joker to a standard deck of cards. Dealing the cards from the top face up, when do you expect the joker to appear? When does the first joker appear when you add N jokers?

3. Jana is driving to work and notices that, on her car's 6-digit odometer, the last 4 digits are palindromic. After one kilometre, the last 5 digits are palindromic. After another kilometre, the middle 4 digits are palindromic. Another kilometre, the all 6 digits are palindromic. What number did Jana start with?

4. (Larson) What is the largest area of a rectangle that can be inscribed in a circle?

5. Can you cover a chessboard with two opposite corners removed with dominoes?

6. When can an $m \times n$ rectangle be covered by the tetromino □□□□?

7. Evaluate the following integral

   (Putnam 1980 A3)
   $$\int_0^{\pi/2} \frac{dx}{1+(\tan x)^{\sqrt{2}}}.$$

8. (Putnam 1998 B2) Suppose $0 < b < a$. Determine the minimum perimeter of a triangle with one vertex (a, b), one vertex on the x-axis, and last vertex on the line $y = x$. You may assume that such a triangle exists.

9. (Putnam 1985 B1) Let k be the smallest positive integer with the following property: There are distinct integers m_1, m_2, m_3, m_4, m_5 such that the polynomial
   $$p(x) = (x - m_1)(x - m_2)(x - m_3)(x - m_4)(x - m_5)$$
   has exactly k non-zero coefficients. Find with proof, a set of integers m_1, m_2, m_3, m_4, m_5 for which this minimum k is acheived.