Abstracts for BIRS Workshop
Symmetries of Graphs and Networks
November 23–28, 2008

Speaker Index

Monday, November 24

9:00–9:50 Dragan Marušič

University of Ljubljana and University of Primorska, Slovenia

Searching for an innovative approach to the Hamilton cycle/path problem in vertex-transitive graphs

In 1969, Lovasz asked whether every finite connected vertex-transitive graph has a Hamiltonian path, thus tying together two seemingly unrelated concepts: traversability and symmetry of graphs. This problem, together with its Cayley graph variation, conjecturing that every connected Cayley graph is hamiltonian, has spurred quite a bit of interest in the mathematical community, producing, amongst other, conjectures and counterconjectures with regards to its truthfulness.

In this talk recent results on this topic will be discussed with a special emphasis given to vertex-transitive graphs of order a product of two primes and to cubic Cayley graphs of groups with a $(2, s, 3)$-presentation. Some possible future directions addressing certain partial questions in the search for Hamilton paths and cycles in vertex-transitive graphs will also be discussed.

10:30–11:20 Brian Alspach

University of Newcastle, Australia

Hamilton paths in vertex-transitive graphs

A graph is Hamilton-connected if for any two vertices u and v there is a Hamilton path whose terminal vertices are u and v. Similarly, a bipartite graph is Hamilton-laceable if for any two vertices u and v from distinct parts there is a Hamilton path with terminal vertices u and v. We present a survey of what is known about Hamilton-connected and Hamilton-laceable vertex-transitive graphs.

2:00–2:50 István Kovács

University of Primorska, Slovenia

S-rings and their applications in graph theory

S-rings were introduced by Schur in order to study permutation groups containing a regular subgroup. In the 80's Klin and Pöschel suggested a scheme based on S-rings over cyclic groups to attack the isomorphism problem of circulant graphs. This scheme was an essential ingredient in a solution of the problem given by Muzychuk.

In the talk we present some techniques of S-rings and their applications in graph theory. In particular, we focus on questions related with automorphism groups of circulant graphs, and the problem of classifying finite CI-groups.

3:45–4:35 Chris Godsil

University of Waterloo, Canada

Association Schemes

Association schemes are significant because they combine interesting combinatorial objects with a rich algebraic structure. They have significant applications in design theory, coding theory, knot theory and in many other areas including, of course, graph theory. My talk will provide a short introduction to association schemes, with an emphasis on their relation to the main themes of this workshop.

Tuesday, November 25

9:00–9:50 Cheryl Praeger

University of Western Australia, Australia

The normal quotient philosophy for edge-transitive graphs

Studying normal quotients has proved an effective way to describe the structure of many families of finite edge-transitive graphs. The normal quotient approach was initiated in my investigation of s-arc transitive graphs, and then refined in collaboration with Giudici and Li to develop our theory of locally s-arc transitive graphs. I will attempt to present the essence of this philosophy - hopefully with reference to a new analysis of an infinite family of edge-transitive graphs.

10:30–11:20 Michael Giudici

University of Western Australia, Australia

Semiregular automorphisms of vertex-transitive graphs

A semiregular automorphism of a digraph is a nontrivial automorphism such that all cycles have the same length. In 1981 Marušič asked if every finite vertex-transitive digraph with at least two vertices has such an automorphism. Klin extended this question to the more general setting of 2-closed groups and this lead to what is today known as the polycirculant conjecture that every finite transitive 2-closed permutation group has a semiregular element. In this talk I will give a survey of the work which has been done on this conjecture.

2:00–2:50 Cai Heng Li

University of Western Australia, Australia

Symmetrical factorisations and decompositions of graphs

I will introduce several new (recently defined) types of combinatorial structures associated with group actions. These structures are related to and generalise various combinatorial objects, such as 2-designs, regular maps, near-polygnal graphs, and linear spaces. One of them is homogeneous factorisations of complete graphs, which is a natural generalisation of vertex-transitive self-complementary graphs. I will discuss relations among these structures, give various interesting and pertinent examples, and present a number of open research problems.

3:30–4:20 Marston Conder

University of Auckland, New Zealand

Combinatorial and computational group-theoretic methods in the study of graphs and maps with maximal symmetry

This lecture will describe a collection of useful aspects of combinatorial group theory and associated computational methods, with special reference to finitely-presented groups and their applications to the study of graphs and maps having maximal symmetry. Recent results include the determination of all arc-transitive cubic graphs on up to 2048 vertices, and of all regular maps of genus 2 to 100, and discovery of some new records in the degree-diameter problem. Moreover, patterns in the results obtained by computation have led to new theorems (in the last two years) about the genus spectra of chiral maps, regular maps with simple underlying graph, regular embeddings of circulants, and regular embeddings of n-cubes Q_n.

Wednesday, November 26

9:00–9:30 Aleksander Malnič

University of Ljubljana, Slovenia

On non-normal arc transitive 4-valent dihedrants

Let X be a connected 4-valent arc-transitive Cayley graph on a dihedral group D_n such that X is bipartite, with the two bipartition sets being the two orbits of the cyclic subgroup within D_n. The following holds: if D_n is not normal in $\mathrm{Aut}(X)$, then X is isomorphic either to the lexicographic product $C_n[2K_1]$ with $n \geq 4$ even, or to one of the five sporadic graphs on $10, 14, 26, 28$ and 30 vertices, respectively.

This is joint work with István Kovács and Boštjan Kuzman.

9:40–10:10 Primož Šparl

University of Ljubljana, Slovenia

On half-arc-transitive metacirculants of valency 4

A *half-arc-transitive graph* is a vertex- and edge- but not arc-transitive graph. In 1966 Tutte observed that such graphs are necessarily of even valency, and so the smallest admissible valency for half-arc-transitive graphs is 4. The class of half-arc-transitive graphs of valency 4 was and still is a very active area of research. However, despite all the efforts we still do not have a complete classification.

It turns out that many of the half-arc-transitive graphs of valency 4 are metacirculants. (Following Alspach and Parsons, a *metacirculant graph* is a graph admitting a transitive group generated by two automorphisms ρ and σ, where ρ is (m, n)-semiregular for some integers $m \geq 1$ and $n \geq 2$, and where σ normalizes ρ, cyclically permuting the orbits of ρ in such a way that σ^m has at least one fixed vertex.) Each half-arc-transitive metacirculant of valency 4 belongs to one (or possibly more) of four classes of such graphs, reflecting the structure of the quotient graph relative to the semiregular automorphism ρ. In this talk we present some results concerning the attempt to classify each of these four classes and study their interconnection.

10:50–11:20 Alison Thomson

University of Melbourne, Australia

Frobenius networks: routing, gossiping and diameter

Let $G = K : H$ be a Frobenius group. A first-kind G-Frobenius graph is a Cayley graph $Cay(K, S)$ such that $S = a^H$ is an H-orbit on K which generates K, where either H has an even order or a is an involution. Such graphs were first studied by Sole in the context of edge-forwarding index followed by an in-depth study by Fang, Li and Praeger. We show that first-kind Frobenius graphs admit perfect routing and gossiping schemes in some sense.

We classify the first-kind Frobenius circulant graphs of valency 4 and give formulas for their diameter and edge forwarding index. Our approach uses elementary number theory and a geometrical representation of such graphs by integer lattices.

Thursday, November 27

9:00–9:50 Edward Dobson

Mississippi State University, U.S.A.

Automorphism Groups of Cayley Graphs

We will first survey results concerning the automorphism group of Cayley graphs, focusing on results which give a description of the automorphism group of Cayley graphs of a given fixed group G. We then discuss additional results concerning special classes of Cayley graphs of a given fixed group G, as well as mentioning possible future areas of research on automorphism groups of Cayley graphs.

10:30–11:00 Pablo Spiga

University of Padova, Italy

CI-groups with respect to ternary relational structures

In this talk, we present some recent results on the classification of CI-groups with respect to digraphs. Also, we extend the classical CI-problem for other types of combinatorial objects and finally we focus on the classification of CI-groups with respect to ternary relational structures.

1:40–2:10 Klavdija Kutnar

University of Primorska, Slovenia

Hamiltonian cycles in cubic Cayley graphs

It was proved by Glover and Marušič, *J. Europ. Math. Soc.* **9** (2007), 775-787, that cubic Cayley graphs arising from groups $G = \langle a, x | a^2 = x^s = (ax)^3 = 1, \rangle$ having a $(2, s, 3)$-presentation, that is, from groups generated by an involution a and an element x of order s such that their product ax has order 3, have a Hamiltonian cycle when $|G|$ (and thus also s) is congruent to 2 modulo 4, and have a Hamiltonian path when $|G|$ is congruent to 0 modulo 4.

In this talk the existence of a Hamiltonian cycle will be shown when apart from $|G|$ also s is congruent to 0 modulo 4, thus leaving $|G|$ congruent to 0 modulo 4 with s either odd or congruent to 2 modulo 4 as the only remaining cases to be dealt with in order to establish existence of Hamiltonian cycles for this particular class of cubic Cayley graphs.

This is joint work with Henry H. Glover and Dragan Marušič.

2:20–2:50 Primož Potočnik

University of Ljubljana, Slovenia

Arc-transitive graphs with large vertex-stabilisers

A classical result by Tutte states that the order of the vertex-stabiliser in a connected trivalent arc-transitive graph cannot exceed $3 \cdot 2^4 = 48$. It is also well known that no such bound exists if the tetravalent graphs are considered rather than trivalent (an obvoius counter-example being the family of the wreath graphs $W_n = C_n[\bar{K}_2]$). However, tetravalent graphs the order of their vertex stabilisers exceed a certain bound, seem to be closely related the family of wreath graphs. In this talk I will report on some recent result proved by Gabriel Verret and myself concerning the structure of tetravalent arc-transitive graphs with very large vertex-stabilisers.

Friday, November 28

9:00–9:30 Shaofei Du

Capital Normal University, P.R. China

Graph Coverings and Lifting Techniques

In this talk, I shall present some methods for constructing voltage graphs by combinatorial tools and by group theoretical tools, and show a linear criteria for liftings of automorphisms when the covering transformation group is elementary abelian. As an application, I shall show a classification of 2-arc-transitive regular covers of complete graphs with the covering transformation group Z_p^3 and a classification of 2-arc-transitive Cayley graphs on dihedral groups.