Solutions to Homework 5 - Math 2000

The solutions to 2.39,2.43,2.45,2.49,,2.53,2.55 may be found in the book.
Exercises 2.46,2.48 (a),(c) were done during the tutorial.

(# 2.40) Write negations of the following open sentences.
(a) Either $x = 0$ or $y = 0$.
(b) The integers a and b are both even.
Solution. (a) Let $P(x), Q(y)$ be the open sentences

$$P(x) : x = 0,$$
$$Q(y) : y = 0.$$

The first open sentence is just

$$P(x) \vee Q(y).$$

It's negation is

$$\sim (P(x) \vee Q(y)) \equiv \sim P(x) \wedge \sim Q(y) \equiv (x \neq 0) \wedge (y \neq 0).$$

In words this is just

'x is not equal to zero and y is not equal to zero.'

or

'x and y are not equal to zero.'

(b) Let $P(a), Q(b)$ be the open sentences

$$P(a) : a \text{ is even},$$
$$Q(b) : b \text{ is even}.$$

The first open sentence is just

$$P(a) \wedge Q(b).$$

It's negation is

$$\sim (P(a) \wedge Q(b)) \equiv \sim P(a) \vee \sim Q(b) \equiv (a \text{ is odd}) \vee (b \text{ is odd}).$$

In words this is just

'a is odd or b is odd.'

or

'a or b is odd.'

(# 2.48) Determine the truth value of each of the following statements.
(b) $\forall n \in \mathbb{N}, n + 1 \geq 2$.
(d) $\exists x \in \mathbb{R}, 3x^2 - 27$.
Solution. (b) This is a true statement. Note that $\mathbb{N}$ is just the set of positive integers $\{1, 2, 3, \ldots\}$. Thus if $n \in \mathbb{N}$ then $n \geq 1$ is true. Thus adding 1 to each side of the inequality $n + 1 \geq 1 + 1 = 2$ is true for all $n \in \mathbb{N}$.
(d) This is a true statement. Note that if $x = 3$ then $3(3)^2 = 3 \cdot 9 = 27$. Thus there exists a real number which satisfies $3x^2 = 27$.