SOLUTIONS QUIZ 1 - MATH 2000 January 22, 2009

Question 1

Determine $\mathcal{P}(\mathcal{P}(\{1\}))$ and determine its cardinality. *Solution*. Note that

$$\mathcal{P}(\{1\}) = \{\phi, \{1\}\}$$

and thus

$$\mathcal{P}(\mathcal{P}(\{1\})) = \{\phi, \{\phi\}, \{\{1\}\}, \{\phi, \{1\}\}\}.$$

Therefore

$$|\mathcal{P}(\mathcal{P}(\{1\}))| = 4.$$

Question 2

Let $U = \{2, 4, 6, 8, 10, 12, 14, 16\}$ be the universal set, $A = \{2, 6, 10, 14\}$, and $B = \{4, 10, 16\}$. Determine the following sets: $A \cup B$, A - B, and \overline{A} .

Solution. We have that

$$A \cup B = \{2, 4, 6, 10, 14, 16\},\$$
$$A - B = \{2, 6, 14\},\$$
$$\overline{A} = \{4, 8, 12, 16\}.$$

Question 3

Which of the following are partitions of $A = \{1, 2, 3, 4, 5, 6, 7\}$?

$$S_{1} = \{\{1, 3, 4\}, \{2, 7\}, \{5\}, \{2, 6\}\}$$

$$S_{2} = \{A\}$$

$$S_{3} = \{\{1, 2, 3, 4\}, \{5, 6\}\}$$

$$S_{4} = \{\{1\}, \phi, \{2, 3, 4\}, \{5, 6, 7\}\}$$

$$S_{5} = \{\{1, 3, 5, 7\}, \{2, 6\}, \{4\}\}$$

Solution. S_2 and S_5 are the only partitions of A. S_1 is not a partition of A since $\{2,7\} \cap \{2,6\} = \{2\}$. S_3 is not a partition of A since $\bigcup_{X \in S_3} \subset A$. S_4 is not a partition of A since $\phi \in S_4$.