

SOLUTIONS QUIZ 2 - MATH 2000
January 29, 2009

Question 1

For the open sentence $P(A) : A \subseteq \{4, 5, 6\}$ over the domain $S = \mathcal{P}(\{3, 4, 5\})$, determine:

- (a). all $A \in S$ for which $P(A)$ is true;
- (b). all $A \in S$ for which $P(A)$ is false;
- (c). all $A \in S$ for which $A \cap \{4, 5, 6\} = \phi$.

Solution. First note that

$$S = \mathcal{P}(\{3, 4, 5\}) = \{\phi, \{3\}, \{4\}, \{5\}, \{3, 4\}, \{3, 5\}, \{4, 5\}, \{3, 4, 5\}\}.$$

- (a). Therefore $P(A)$ is true if A is one of:

$$\phi, \{4\}, \{5\}, \{4, 5\}.$$

- (b). Likewise $P(A)$ is false if A is one of:

$$\{3\}, \{3, 4\}, \{3, 5\}, \{3, 4, 5\}.$$

- (c). Lastly $A \cap \{4, 5, 6\} = \phi$ if A is one of:

$$\phi, \{3\}.$$

Question 2

Consider the statements P : 13 is prime. and Q : 27 is prime. Write each of the following statements in words, and indicate whether it is true or false.

- (a). $\sim P$
- (b). $P \vee Q$
- (c). $P \wedge Q$
- (d). $(\sim P) \wedge Q \implies Q$

Solution.

Note that P : 13 is prime. is true and Q : 27 is prime. is false.

- (a). $\sim P$ is the statement '13 is not prime.' $\sim P$ is false.
- (b). $P \vee Q$ is the statement '13 is prime or 27 is prime.' $P \vee Q$ is true.
- (c). $P \wedge Q$ is the statement '13 is prime and 27 is prime.' $P \wedge Q$ is false.
- (d). $P \implies Q$ is the statement

'If 13 is not prime and 27 is prime, then 27 is prime.'

or

'13 is not prime and 27 is prime implies 27 is prime.'

Note that $\sim P$ is false and Q is false. Thus $\sim P \wedge Q$ is false. Since Q is false, then it follows that $\sim P \wedge Q \implies Q$ is true.