SOLUTIONS QUIZ 3 - MATH 2000

Question 1

For statements P and Q, show that $(\sim Q) \implies (P \land (\sim P))$ and Q are logically equivalent.

	P	Q	$\sim Q$	$\sim P$	$P \land (\sim P)$	$Q \implies (P \land (\sim P))$
	Т	Т	F	F	F	Т
Proof.	Т	F	Т	F	\mathbf{F}	F
	F	Т	\mathbf{F}	Т	\mathbf{F}	Т
	F	F	Т	Т	\mathbf{F}	F

Question 2

Give the definition of a contradiction.

Solution. A contradiction is a compound statement which is false for all possible combinations of truth values of the component statements that form S.

Question 3

Consider the open sentences

$$P(x): (x+2)(x-3) = 0$$
 and $Q(x): x^2 = 4$

over the domain $S = \{-2, 0, 2, 3\}.$

(i) State in words the open sentence $P(x) \Leftrightarrow Q(x)$.

(ii) Determine the truth value of $P(x) \Leftrightarrow Q(x)$ for all values of $x \in S$. Solution. (i)

'(x+2)(x-3) = 0 is equivalent to $x^2 = 4$.'

	x	P(x)	Q(x)	$P(x) \Leftrightarrow Q(x)$
	-2	0=0, T	4 = 4, T	Т
(ii)	0	-6=0, F	0=4, F	Т
	2	-4=0, F	4=4, T	F
	3	0=0, T	$9{=}4, F$	F