SOLUTIONS QUIZ 4 - MATH 2000

Question 1

For statements P, Q, and R, list the 8 fundamental logical equivalences. There are 2 commutative laws, 2 associative laws, 2 distributive laws, and 2 De Morgan's laws.

Solution. Please consult the book page 49, Theorem 2.18.

Question 2

Consider the following quantified statement:

P: 'There exist rational numbers x and y such that $x-y \leq -7$ and xy > 21.'

(i) Using quantifiers express in symbols the quantified statement P.

(ii) Express in words the negation of this quantified statement.

(iii) Using quantifiers express in symbols the negation of the quantified statement P.

Solution. (i)

$$\exists x \in \mathbb{Q}, \exists y \in \mathbb{Q}, x - y \leq -7 \land xy > 21.$$
(ii) 'For all rational numbers x and y, $x - y > -7$ or $xy \leq 21$.'

or

'For every rational number x and y, x - y > -7 or $xy \le 21$.'

(iii)

$$\forall x \in \mathbb{Q}, \forall y \in \mathbb{Q}, x - y > -7 \lor xy \le 21.$$