

Department of Mathematics and Computer Science

Probability and Statistics Challenge Problems 1

Solution

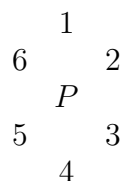
Chance at Freedom

1. In order to amuse the cruel king a prisoner is placed in a circular room with six doors around the room numbered clockwise 1 through 6. He is told that behind two consecutive doors (ie 1,2 or 2,3 or 3,4 or 4,5 or 5,6 or 6,1) are lions. If the prisoner is able to open two doors without releasing a lion, he will be set free. Otherwise he will be lunch. However, there is a catch. His first choice depends upon the roll of a die. If he survives his first choice, for his second choice he may either select the next door in line (eg if he rolled a 3 first he may subsequently choose door 4), or he may roll the die again.

Now suppose he is successful with his first door. That is he rolls the die, opens the door according to the number rolled and there is no lion behind the door. For his next choice, should he select the next door in line, or should he roll the die again, and why?

Note that it is permissible for the same door to be opened twice. For example, if the prisoner rolls a 3 to begin and opens door 3 successfully he may either choose door 4 as his second choice or roll the die again in order to select another door. He is allowed to roll another 3 which of course would mean he was successful again. You should also assume that the die is fair. Each number is equally likely.

The room looks a bit like this, with P representing the prisoner.



The prisoner's best choice is to select the next door in line. Since the prisoner was successful with his first choice, he selected one of the four safe doors. Now only one of the four safe doors has a bad door next in line, so by selecting the next door in line he has a $3/4$ chance of opening another safe door and a $1/4$ chance of opening a door with a lion behind. For example, suppose lions are behind door 1 and door 2. Since the prisoner's first choice was successful, he must have selected door 3, or door 4, or door 5, or door 6. Of these four, only door 6 will cause a problem if he selects the next door in line as his second choice.

Meanwhile, if he chooses instead to roll the die again in order to select the next door, he has $4/6$ chance of opening a safe door and a $2/6$ chance of opening a door with a lion behind.

Therefore he has a better chance at freedom if he selects the next door in line.