ABSTRACTS

Fourth Prairie Discrete Mathematics Workshop
University of Lethbridge
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On the hamiltonicity of generalized Petersen graphs

Brian Alspach
University of Regina

The generalized Petersen graph $\mathrm{GP}(n,k)$ has vertex set $\{u_1, u_2, \ldots, u_n, v_1, v_2, \ldots, v_n\}$ and edge set $\{u_iu_{i+1}, v_jv_{j+k}, u_i, v_i :, 1 \leq i, j \leq n\}$, where subscript arithmetic is carried out modulo n. The problem of classifying the hamiltonian generalized Petersen graphs was completed in the 1980s. One of the problems Jiping considered in his Ph.D. thesis was what one can say about Hamilton connectivity and Hamilton laceability of generalized Petersen graphs. This talk will concentrate on his work in this direction.

Decomposition Theorem and Switch Box Design

Hongbing Fan
Wilfrid Laurier University

Since 1999 Jiping had been involved in the research on switch box design for reconfigurable interconnection networks. He made a solid contribution to the switch box design methodology, including graph modelling, decomposition theorems on routing requirements, as well as some concrete designs and routing algorithms. In this talk, I will present some of these contributions including the proof of $f_2(k) = (k+3-i)/3, i = k(mod\ 6)$ and $1 \leq i \leq 6$, where $f_2(k)$ denotes the maximum degree of non-decomposable regular 2-graphs (multiple hypergraphs with edge size at most 2) on k vertices, and the design of (k,w)-universal switch boxes.

The linguistics of subtraction games

Richard Guy
University of Calgary

Subtraction games are special cases of Nim, and should therefore be childishly simple to play and to analyze, and yet there are many problems which still seem to baffle us.

Pivot and Gomory Cut

Ryan Hayward
University of Alberta

We introduce a feasibility heuristic for 0-1 mixed integer programs that integrates fractional Gomory cuts into the simplex-based pivoting framework of the "Pivot and Complement" heuristic of Balas and Martin. Our heuristic is easily implemented and, on a standard test suite compiled from MIPLib and other sources, performs comparably to the best known feasibility heuristics. Moreover, on a randomly generated set of feasibility-hard market share instances analogous to the optimality-hard market share instances of Cornuejols and Dawande, our heuristic significantly outperforms both Cplex and the "Feasibility Pump" heuristic of Fischetti and Lodi.

This is joint work with Shubhashis Ghosh.

Isofactorization of circulant graphs

Don Kreher
Michigan Technological University

A graph has a $k-$isofactorization just when its edges can be decomposed into isomorphic $k-$edge subgraphs. The problem of showing that every circulant graph has a $k-$isofactorization, whenever k divides the number of edges, was first posed by Alspach in 1982. Recent progress on this problem will be presented. This joint work with Brian Alspach and Danny Dyer and also with Erik Westlund.

The Firefighter Problem for Trees and Graphs

Gary MacGillivray
University of Victoria

We discuss a discrete time problem that was introduced by Bert Hartnell in 1995. Imagine that, at time 0, a fire breaks out at a vertex of a graph G. At each subsequent time $t = 1, 2, \ldots$, the firefighter "defends" a vertex of G and then the fire spreads from each "burning" vertex to all of its undefended neighbours. Once a vertex is burning or defended, it remains so from then onwards. The process terminates when the fire can no longer spread. The main problem is to determine the maximum number of vertices that can be saved from burning, though a number of variations have received attention. We will survey questions, approaches and solutions that have arisen over the past 11 years.

The Cayley Isomorphism Problem

Joy Morris
University of Lethbridge

In a perfect world in which all isomorphisms between graphs must be in some sense "natural," it would be possible to attack the problem of determining whether or not given graphs are isomorphic, simply by checking a (hopefully small) class of "natural" isomorphisms. For Cayley graphs, "natural" isomorphisms between the graphs $\mathrm{Cay}(G;S)$ and $\mathrm{Cay}(G;S')$ on the group G, would consist exclusively of automorphisms of the group G.

Alas, our world is not perfect. However, there are some Cayley graphs $X = \mathrm{Cay}(\mathrm{G};\mathrm{S})$ for which the isomorphism problem can be solved in this manner. That is, for such a graph X, the Cayley graph $\mathrm{Cay}(G;S')$ is isomorphic to X if and only if there is an automorphism of G that takes S to S' (and hence acts as a graph isomorphism). Such a graph is said to have the Cayley Isomorphism, or CI, property. Furthermore, there are some groups G for which every Cayley graph $\mathrm{Cay}(G;S)$ has the CI property; these groups are said to have the CI property. The Cayley Isomorphism problem is the problem of determining which graphs, and which groups, have the CI property.

This talk will present an overview of the Cayley Isomorphism problem and the known results.

Computational techniques in the construction of combinatorial objects

B. Tayfeh-Rezaie
Institute for Studies in Theoretical Physics and Mathematics, Tehran, Iran

Since their advent, computers have increasingly been used in the construction and classification of combinatorial objects. Although there are general methods which are applicable to a vast variety of combinatorial objects, for any specific class of objects one may find some techniques which dramatically decrease the time of computation. In this talk, we review some of these methods in the construction of combinatorial designs. The objects of interest include block designs, orthogonal designs, Hadamard matrices, Williamson matrices, complementary sequences and so on.

Transversals and Defining Sets in Rectangles and Latin Squares

G. H. J. van Rees
University of Manitoba

First Topic: A greedy Defining Set is a set of symbols in a Latin Square with the property that when the square is systematically filled in with a greedy algorithm, the greedy algorithm succeeds. Let g_n be the smallest defining set for any Latin Square of order n. We give theorems on the upper bounds of g_n and a table listing upper bounds of g_n for small values of n. For a circulant Latin square, we find that the size of the smallest Greedy Defining Set is $\lfloor n(n-1)/6 \rfloor$.

Second Topic: For $2 \leq m \leq n$, we investigate what the largest symbol frequency can be so that any $m \times n$ array must contain a transversal (m cells, one in each row and at most one in each column such that the symbols in the cells are distinct). We give the asymptotic value and consider the more interesting square or almost square cases.

Third Topic: If time, recent results in critical sets and the proof of the existence of Bachelor Latin Squares will be given.

There are many open problems in this area of research.

Permutation Polynomials: Applications and Constructions

Steven Wang
Carleton University

A polynomial f over a finite field $\mathbb{F}_q$ (or a finite ring R) is called a permutation polynomial of $\mathbb{F}_q$ (or R) if the mapping f permutes the elements of $\mathbb{F}_q$ (or R). Permutation polynomials were first investigated by Hermite, and since then, many studies concerning them have been devoted. In the last 20 years there has been a revival in the interest for permutation polynomials, in part due to their applications in combinatorics and cryptography. In this talk I will give a brief introduction to these applications of permutation polynomials and then describe some new classes of permutation polynomials of finite fields.

Recent progress on the cage problem

Qinglin Roger Yu
Thompson Rivers University

A $(k;g)-$cage is a $k-$regular graph of the girth g with the least number of vertices. In this talk, we present a brief review of the history of the cage problem and then the recent progress on its connectivity, degree monotonicity, and factorial properties.

Edge disjoint spanning trees, dense subgraph partition, graph clustering

Cun-Quan Zhang
West Virginia University

Let $G = (V,E)$ be a graph (or multi-graph) and H be a subgraph of G. The dynamic density of H is the greatest integer k that $\min_{\forall \mathcal{P}}\{|E(H/\mathcal{P})|/(|V(H/\mathcal{P})|-1)\} > k$ where the minimum is taken over all possible partitions $\mathcal{P}$ of the vertex set of H, and $H/\mathcal{P}$ is the graph obtained from H by contracting each part of $\mathcal{P}$ as a single vertex. A subgraph H of G is a level-k community if H is a maximal subgraph of G with dynamic density at least k. An algorithm is designed in this paper to detect all level-h communities of a given multi-graph $G = (V,E)$ ($n = |V|$, $m = |E|$) with the complexity is $O(n^2h^2)$. (U.S. Patent pending: # 60/677,655)

Application 1 – Dense subgraph clustering: Clustering is one of the most important tools in statistics. In a graph theory model, the clustering processing is to find all denser subgraphs of an input graph. Dynamic density is one of few well-defined measuring for graph density. Furthermore, this clustering method is one of few available graph theoretical methods that are mathematically well-defined, supported by rigorous mathematical proof and able to achieve the optimization goal with polynomial complexity.

Application 2 – Disjoint spanning trees: By applying Tutte-Nash-Williams Theorem, it is easy to prove that the dynamic density of H is at least k if and only if, for every edge e_0 of H, $H \setminus \{e_0\}$ contains k edge-disjoint spanning trees. Therefore, the algorithm can be applied to find the maximum number of edge-disjoint spanning trees in a multi-graph $G = (V,E)$ ($n = |V|$, $m = |E|$, W be the multiplicity). This optimization problem has been extensively studied and a long list of algorithms have been introduced with the following complexity: (Cunningham (1984): $O(nm^8)$; Cunningham (1985): $O(nm)\phi_M$ or $O(n^4m)$; Gabow (1991): $O(n^4m^2\log^2 W)$; Gusfield (1991): $O(n^3m)$; Banahona (1995): $O(n^2)\phi_M$; Gabow (1998): $O(n^2m\log(n^2/m))$; Barahona (2004): $O(n)\phi_M$; *etc.* Note: $\phi_M = O(nm\log(n^2/m))$ is the complexity of the min-cut max-flow problem (by Goldberg and Tarjan 1988).

Application 3 – Disjoint parity subgraphs: With the main theorem, the following conjecture is solved: if the odd-edge-connectivity of a graph G is $2k+1$ then G contains k edge-disjoint parity subgraphs.

(Joint work with Y. B. Ou.)