ABSTRACTS

The Graph Theory of Brian Alspach
Simon Fraser University
Burnaby, B.C.
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In multi-author talks “$\star$” indicates the speaker.

Invited Talks

The Erdős-Pósa property for long circuits

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Let G be a graph and $\mathcal{F}$ a family of graphs. A *transversal* of $\mathcal{F}$ is a set X of vertices of G such that $G - X$ contains no member of $\mathcal{F}$. The family $\mathcal{F}$ is said to have the *Erdős-Pósa property* if there exists a function $f : \mathbb{N} \to \mathbb{N}$ such that every graph G contains either k disjoint members of $\mathcal{F}$ or a transversal of $\mathcal{F}$ of size at most $f(k)$. This concept originated in a paper of P. Erdős and L. Pósa, who established the existence of such a function f when $\mathcal{F}$ is the family of circuits.

We outline a proof that, for every l, the family $\mathcal{F}_l$ of circuits of length at least l satisfies the Erdős-Pósa property, with $f(k) = O(lk^2)$. This sharpens a result of C. Thomassen, who obtained a doubly exponential bound on $f(k)$. Applying a result of E. Birmelé, we obtain as a corollary that graphs without k disjoint circuits of length l or more have tree-width $O(lk^2)$. We also discuss a bound on the tree-width of graphs containing no large prism minor.

This work was motivated by the theorem of N. Robertson and P.D. Seymour that a graph containing no large grid minor has bounded tree-width, our aim being to reduce their bound.

On groups of odd prime-power degree that contain a full cycle

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Burnside has proven that a transitive group of prime degree p is either doubly-transitive or contains a normal Sylow p-subgroup. We will discuss applications of this result to the study of vertex-transitive graphs. Additionally, we will discuss recent extensions of Burnside's result as well as applications of these extensions to the study of vertex-transitive graphs.

Explaining Youngs' bimodality phenomenon for embedded graphs

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In 1996, Youngs showed that every loopless quadrangulation of the projective plane has chromatic number either 2 or 4, but never 3. (A *quadrangulation* is an embedded graph with all faces having length 4.) This surprising 'bimodal' behaviour of chromatic number had not before been observed for any naturally defined class of graphs. Is Youngs' observation a curious singularity?

The answer is both "yes" and "no". We show that Youngs' phenomenon is common among certain families of embedded graphs, but with two provisos. First, other than Youngs' example, bimodality is undetectable without the use of the refined notion of "circular chromatic number." Second, for higher surfaces, one must assume that the *edge-width* of the embedded graph is large. That is, all noncontractible circuits must be long.

Here are some results arising from this work:

- Bimodal behaviour of circular chromatic number occurs among triangulations of any surface, and among even-faced graphs on non-orientable surfaces, provided the edge-width is high.
- For even-faced graphs on the projective plane, an easily calculated formula for circular chromatic number is given.
- Tutte's flow-colouring duality of plane graphs holds "approximately true" for any embedded graph of high edge-width.
- Group-valued tensions on embedded graphs can be very sensitive to the group's interaction with the first (integer) homology group of the surface.

Techniques include the use of Robertson-Seymour structure theory, and the total-unimodularity of network matrices. Most of this is joint work with M. DeVos, B. Mohar, D. Vertigan and X. Zhu.

Colouring Interesting Graphs

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Much attention has been devoted to colouring problems on large classes of graphs. But there are many interesting special graphs for which we would like to have more information about there chromatic numbers, and related parameters. Among the examples I have in mind, are generalisations of the Kneser graphs, and Cayley graphs for abelian groups.

In my talk I will explain why these graphs deserve our attention, and I will describe some recent progress.

And now for something somewhat different (what you may not know about BA)

Gena Hahn

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A reminder of some things, mathematical or not, that Brian Alspach is perhaps not known for.

Homomorphisms of graphs with bounded degrees

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I will present recent results, some joint with Huang Jing and Tomas Feder, others joint with Jarik Nesetril, on the complexity of homomorphism problems restricted to graphs with bounded degrees. A conjecture of Dyer and Greenhill proposes that degree restrictions do not change the classification of the complexity of counting homomorphisms. On the other hand, results of Galluccio, Haggkvist, Nesetril, and the speaker indicate that degree restrictions do yield interesting new polynomial cases for the basic homomorphism problem. The main results I will present state that for list homomorphisms, degree restrictions do not help. I will also give examples, for homomorphism extension problems, where the number of new polynomial cases depends on how much the degrees are restricted. Implications for constraint satisfaction problems, with restrictionson the number of times a variable can be used, will also be discussed.

Connected Rigidity Matroids and Unique Realizations of Graphs

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A d-dimensional *framework* is a straight line embedding of a graph G in $\mathbb{R}^d$. We shall only consider *generic* frameworks, in which the co-ordinates of all the vertices of G are algebraically independent over $\mathbb{Q}$. Two frameworks for G are *equivalent* if corresponding edges in the two frameworks have the same length. A framework is a *unique realization* of G in $\mathbb{R}^d$ if every equivalent framework can be obtained from it by a rigid congruence of $\mathbb{R}^d$. Bruce Hendrickson proved that if G has a unique realization in $\mathbb{R}^d$ then G is $(d+1)$-connected and 'redundantly rigid'. He conjectured that every realization of a $(d+1)$-connected and redundantly rigid graph in $\mathbb{R}^d$ is unique. This conjecture is true for $d = 1$ but was disproved by Bob Connelly for $d \geq 3$. We resolve the remaining open case by showing that Hendrickson's conjecture is true for $d = 2$. As a corollary we deduce that every realization of a 6-connected graph as a 2-dimensional generic framework is a unique realization. Our proof is based on a new inductive characterization of 3-connected graphs whose rigidity matroid is connected. This is joint work with Tibor Jordán.

Degree estimates for the circumference of graphs

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By a classical result of G. A. Dirac, a longest cycle C in a 2-connected graph G with minimum degree $\delta(G)$ has length at least $2\delta(G)$ or is Hamiltonian. Another old result for 3-connected G says that C is dominating (i.e. $G - V(C)$ is independent) or $|C| \geq 3\delta(G) - 3$. We elaborate on some results in this vein, in particular on estimates of the form $|C| \geq 4\delta(G) - 8$ and the rather big exceptional class concerning the estimate $|C| \geq 3\delta(G) - 3$ when C is not dominating and G is merely 2-connected.

A brief history of embedding partial 4-cycle systems

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This is a brief history of the embedding of partial 4-cycle systems starting with "really big" to the best result to-date ... a partial 4-cycle system of order n can be embedded in a 4-cycle system of order at most $2n + 15$.

A Characterization of Pancyclic Complements of Line Graphs

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Let $G = (V, E)$ be a simple graph. The complement of the line graph of G, denoted by $\overline{L(G)}$, has vertex set E, two vertices e_1 and e_2 are adjacent in $\overline{L(G)}$ if e_1 and e_2 are not incident in G. We characterize graphs for which the complements of their line graphs are pancyclic. This talk also includes results that characterize graphs for which the complements of their line graphs have any of the following properties: Hamiltonian, traceable, Hamilton-connected, and Hamilton-laceable. These characterizations lead to linear recognition algorithms too.

Symmetry in Graphs — Some Open Problems

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In this lecture I will discuss some of my favourite open problems in vertex-transitive graphs related to the work of Brian Alspach.

In particular, I will concentrate on the problem of classifying vertex-transitive graphs of specific orders and on the hamiltonicity problem in vertex-transitive and Cayley graphs. I will wrap up this talk by giving an account of graphs ejoying special types of symmetry, such as half-arc-transitive graphs.

Hamiltonian paths and cycles in vertex-transitive graphs and digraphs

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It was conjectured, more than 30 years ago, that every vertex-transitive graph has a hamiltonian path, and that every Cayley graph has a hamiltonian cycle (unless the graph is disconnected). This talk will survey the progress that has been made on these problems, both of which remain very much open. For example, it is still not known whether every Cayley graph on every dihedral group has a hamiltonian cycle. (The cubic case was settled by Brian Alspach and C.-Q. Zhang.) Related questions on directed graphs will also be discussed; for example, it is easy to see that every (connected) circulant digraph has a hamiltonian path, but we do not know which circulant digraphs have hamiltonian cycles.

On Automorphisms of Circulant Graphs

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This talk presents a variety of results on the automorphism groups of circulant graphs, that help us to better understand their structure. It culminates in the presentation of a new polynomial-time algorithm that determines the automorphism group of a circulant graph on a square-free number of vertices; this algorithm is joint work with Ted Dobson of Mississippi State University.

Tournaments in which every arc is in a Hamiltonian path

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One of the earliest results by B. Alspach is that every arc of every regular tournament is contained in cycles of all possible lengths (Canad. Math. Bull. 10 (1967), 283–285). That is, regular tournaments are arc-pancyclic. O. S. Jacobsen proved that every arc of every almost regular tournament is contained in cycles of all possible lengths, except perhaps length 3 (Ph. D. Thesis, Aarhus University, 1972). Subsequently, several other results have appeared in the literature on arc-pancyclic tournaments (or nearly arc-pancyclic). In this talk we discuss tournaments in which every arc is in a Hamiltonian path. Tournaments which are 3-connected have this property. Indeed, C. Thomassen showed that every arc in a 3-connected tournament is in a Hamiltonian cycle (JCT B 28 (1980), 142–163). However, tournaments in which every arc is in a Hamiltonian path need not be strong. We settle a conjecture of L. Quintas concerning digraphs in which every arc is in a Hamiltonian path. We discuss this property for a special family of tournaments obtained from the transitive tournament by reversing the arcs in a path from transmitter to receiver. And, we also discuss some structure of tournaments that contain an arc which is in no Hamiltonian path.

Extended Petersen graphs

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We discuss some properties of yet another class of graphs whose smallest member is the Petersen graph. These graphs, which we call extended Petersen graphs, arise naturally in the context of a construction of Steiner systems $S(2,4,v)$ with maximal arcs but seem to be interesting on their own.

Variations on Hamiltonian Themes

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The hunt for hamiltonian cycles in graphs started in the 18-th century. Most probably Euler's search for a knight tour of the chess board was the first hamilton cycle problem considered. Nowadays we are flooded by theorems, conjectures, refuted conjectures, web sites whose common characteristic is that if a graph satisfies P then it is Hamiltonian and what not.

It is well known that the problem of finding a hamilton cycle in a graph is "certifiably" difficult. As common among Mathematician, when the going gets rough, the rough "change" the problem. Current trends are to "refine" (mutilate) the definition of hamilton cycle. The first minimal modification was to replace hamilton cycle by hamilton path. A more drastic modification is to allow vertices to be visited more than once. A 2-walk is a closed walk in a graph in which every vertex is visited but not more than twice. A 3-tree is a tree with maximum degree 3. Clearly, if G is hamiltonian, it has a 2-walk but not vice versa. Thus, instead of searching for hamiltonian cycles, several researchers, like M. Ellingham, N. Wormald, F. Ruskey and others, proposed searching for 2-walks, or even k-walks. These modifications create a hierarchy among graph properties:

$$\text{hamiltonian} \subset \text{traceable} \subset \text{2-walk} \subset \text{3-tree} \subset \text{3-walk} \ldots$$

In our work, we chose to retain the hamilton cycle and modify the graph. Instead of searching for hamilton cycles in G we look for hamilton cycles in the prism over G. This property is "sandwiched" between "hamiltonian" and 2-walk. With this in mind, we revisit many theorems, conjectures, complexity problems etc. and test them for hamilton cycles in the prism over G.
In this talk I'll survey a sample of problems and results of this nature, including some conjecture of Brian and myself dated back to the previous millennium.

This is joint work with: Daniel Král' (Charles University, Prague); Tomas Kaiser and Zdenek Ryjacek (Western Bohemia, University, Pilzen); Heintz-Juergen Voss (Technical University, Dresden).

Subholomorhic Cayley Graphs

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By definition, there are Cayley graphs $G = Cay(A,S)$ for which the stabiliser of the identity of A (as a vertex of G) coincides with the stabiliser of S in the automorphism group of A. Graph isomorphisms between such graphs turn out to be twisted isomorphisms between the underlying groups, allowing in some interesting cases to determine all subholomorphic representations of a given Cayley graph. (Joint work with J. Fournier.)

Cycle decompositions: The past, present, and future

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This talk will be a survey on cycle decompositions of complete and nearly complete graphs, and a tribute to Brian Alspach's enormous impact on research in this area. We shall outline certain aspects of the fascinating history of cycle decompositions, introduce several variants of the problem, describe some of the recent results, and list the open problems.

Totally Magic Graphs

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Given a graph G, a total labeling of G is a one-to-one map from $V(G) \cup E(G)$ to $\{1,2,\dots,|V(G) \cup E(G)|\}$. A total labeling λ is called *vertex magic* if there is a constant h such that, for every vertex x of G, the sum of $\lambda(x)$ and all the $\lambda(xy)$ for y adjacent to x equals h, and *edge magic* if there is a constant k such that, for every edge xy of G, $\lambda(x) + \lambda(xy) + \lambda(y) = k$. A graph is *totally magic* if it has a labeling that is simultaneously vertex magic and edge magic. We shall discuss the existence of totally magic graphs.

Chords of longest circuits in 3-connected graphs

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It was conjectured by Thomassen (1985) that every longest circuit of a 3-connected graph must have a chord. This talk is to survey the partial results to this conjecture and various methods applied in those proofs.

Contributed Talks

Improved Algorithm for Finding the Maximum Clique in a Graph using BDDs

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A new approach for the solution of the maximum clique problem for general undirected graphs was presented in the paper titled "Finding the Maximum Clique in a Graph Using BDDs", by F. Corno, P. Prinetto and M. Sonza Reorda, in 1993. The approach is based on computing the characteristic function of all the completely connected components in the graph, and then finding the maximum cost satisfying assignment of such a function. The novelty of the method is in the use of Binary Decision Diagrams (BDDs) for representing and manipulating characteristic functions. However, we observe that their algorithm need not always produce the right answer. The main contribution of this paper is another algorithm which overcomes this inadequacy in the algorithm of Corno et al.

Banana Trees are Graceful, and why this is easy to show using adjacency matrices

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Banana Trees consist of a star K(1,n) each of whose n endvertices are identified with one of the endvertices of another star. (Consequently, the diameter of these trees is six.)

When the adjacency matrix for each of the gracefully numbered stars is written in a canonical form, it is easy to see how to combine the matrices to produce a graceful numbering for the banana tree.

Other results from matrix constructions will be presented as time allows.

Mutually embeddable vertex-transitive graphs and trees

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Two graphs are *mutually embeddable* or *me* if they are isomorphic to induced subgraphs of each other. While any two finite me graphs are isomorphic, there are many examples of pairwise me non-isomorphic infinite graphs. Closely related to me graphs are the *universal* graphs, which embed all graphs of smaller orders. An example of a universal graph that has attracted much attention is the infinite random graph, R. The graph R is vertex-transitive, and this fact motivates the following problem: find large (say, of cardinality $2^{\aleph_0}$) families of pairwise me non-isomorphic vertex-transitive countable graphs. We will give examples of such families by using the weak Cartesian product and its unique factorization properties. Our techniques give large families of me universal graphs for all infinite cardinals.

We conjecture that if a tree T (of any infinite order) is me with another non-isomorphic one, then T is me with an *infinite* family of pairwise non-isomorphic trees. We will discuss why the conjecture is true in the case of *rayless* trees; that is, trees that contain no infinite path as an induced subgraph.

Cubic inflation, mirror graphs, and Platonic graphs

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Mirror graphs are a common generalization of even cycles and hypercubes. A connected graph $G = (V,E)$ is called a mirror graph if there exists a partition $\mathcal{P} = \{E_1, E_2, \ldots, E_k\}$ of E such that for all $i \in \{1,\ldots,k\}$ the following holds:

(i) $G - E_i$ consists of two connected components G_1^i, G_2^i, and

(ii) there is an automorphism α_i of G that maps G_1^i isomorphically onto G_2^i and every edge of E_i is invariant under α_i.

Mirror graphs are bipartite, vertex-transitive, and closed for Cartesian products. Cubic inflation is an operation that transforms a plane graph into a cubic plane graph; its result can be described as the dual of the barycentric subdivision of a plane graph.

In this talk we present a characterization of (plane) mirror graphs that can be obtained by cubic inflation. There are only five such graphs, namely the inflated Platonic graphs. In turn, a characterization of Platonic graphs involving a partition of a graph into special straight-ahead walks is derived. As an application of both concepts, some new regular isometric subgraphs of hypercubes are constructed.

Extending the Critical Theorem

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The Critical Theorem, due to Crapo and Rota (1970), is one of the fundamental results concerning generalised colouring problems, and has been generalised in a number of different ways. We present a general form of the Critical Theorem that encompasses several of these results. Applications include generalisations of a theorem by Greene (1976) stating how the weight enumerator of a linear code is determined by the Tutte polynomial of the associated vector matroid, as well as generalisations of the MacWilliams identity for linear codes.

List Partitions of Graphs

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The k-partition problem is: Given a graph G and a positive integer k, partition the vertices of G into at most k sets, $A(1)$, $A(2)$, $\ldots$, $A(k)$, where some of the sets may be required to be stable sets and some may be required to induce complete graphs, and some pairs $A(i)$, $A(j)$ may be required to be completely joined and some pairs may be required to not be joined at all, or decide that no such partition exists.

The list k-partition problem is the same problem with the added constraint that each vertex v is given a list $L(v)$ of which of the sets $A(i)$ of the partition it can be in.

Several well-known problems can be formulated as k-partition problems: clique-cutset, stable cutset and 3-colourability are 3-partition problems; 2-clique cutset and skew partition are 4-partition problems.

We have classified the list 4-partition problems as either polytime-solvable or NP-complete, with a single exception. Previously, Feder, Hell, Klein and Motwani had classified the list 3-partition problems as polytime-solvable or NP-complete, and the list 4-partition problems as "quasipolynomial" or NP-complete. De Figueiredo, Klein, Kohayakawa and Reed had given a polytime algorithm for the skew partition problem.

Isomorphic Factorization of the Complete Graph into Cayley Digraphs

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Let Z_p denote the cyclic group of order p where p is a prime number. Let $X = X(Z_p, H)$ denote the Cayley digraph of Z_p with respect to the symbol H. We obtain a necessary and sufficient condition on H so that the complete graph on p vertices can be edge-partitioned into three copies of Cayley digraphs of the same group Z_p each isomorphic to X. Based on this condition on H, we then enumerate all such Cayley graphs and digraphs.

Tandem-win Graphs

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In this version of the Cops and Robber game, the cops move in tandems, or pairs, such that they are at distance at most one from each other after every move. We present a recognition theorem for tandem-win graphs, and a characterization of triangle-free tandem-win graphs.

Regular Cayley maps for finite groups

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For the purposes of this talk, a *regular Cayley map* is a 2-cell embedding of a Cayley graph $C(G,S)$ of a group G (with respect to some generating set S) into an orientable surface, such that the group of all incidence- and orientation-preserving symmetries of the embedded graph is sharply transitive on the arcs of the graph. In particular, the group G acts regularly on vertices of the map, and is complementary to a cyclic group of symmetries fixing a given vertex. Such objects are closely related to finite quotients of $(2, p, q)$ triangle groups, and may also be studied in terms of so-called *skew-morphisms* of the group G.

These concepts will be explained, and illustrated in the special case where the group G is abelian. A number of questions and answers will be given, including some pathological examples, along with a recent spin-off concerning the structure of groups expressible as a product AB of two subgroups A and B where A is abelian and B is cyclic.

This is joint work with Robert Jajcay and Tom Tucker (on regular Cayley maps) and Marty Isaacs (on products of cyclic and abelian subgroups).

Structure Graphs of Complete Cayley Graphs

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Structural results for complete Cayley graphs are found via structure graphs whose vertices are labelled by the equivalence classes of their equally multicolored K_4's. These classes are interpreted as complete edge-connected systems of such K_4's. This yields an infinite family of connected labelled graphs whose diameters are asymptotically of the order of the cubic root of their vertex-set cardinalities. The infinite family is obtained via modular reduction from a structure graph arising from the Cayley graph of the group of integers with the natural numbers taken as generators. This structure graph, of maximum degree 6, has symmetry properties locally involving beehive-type subgraphs and representable via tessellations of regular hexagons and triangles.

On some Oberwolfach-type Problems

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Any 2-factorization of a regular graph is an *Oberwolfach-type* problem. We report on some recent such problems. In particular, we report on the following result of Cavenagh, El-Zanati, Khodkar and Vanden Eynden: Let p be an odd prime and let $e_1, e_2, \ldots, e_n$ be a sequence of nonnegative integers such that the first non-zero term is not one and $\sum_{i=1}^{n} e_i = (p^n - 1)/2$. It is shown that the complete graph P_{p^n} can be decomposed into e_1 C_{p^n}-factors, e_2 $C_{p^{n-1}}$-factors, $\ldots$, and e_n C_p-factors.

The Decomposition of Regular Hypergraphs and Switch Box Designs

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We study the problem of decomposing a regular multiple hypergraph into minimal (non-decomposable) regular hypergraphs. This problem is crucial to the design of switch boxes of circuit switching networks. The problem is intractable in general, but tractable when the number of vertices is fixed. To derive such an algorithm, we use the set of minimal regular hypergraphs, which is computed by solving the Hilbert basis of a system of linear Diophantine equations. Consequently an upper bound for the maximum degree $f(n)$ of minimal regular hypergraphs of n vertices is obtained.

Graceful Graph Labelings and Designs with Special Properties

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It has long been known that block designs can be constructed from graceful labelings of graphs. In this talk it will be demonstrated that some recent powerful constructions, most notably that due to Buratti and Zuanni enable one to utilize graceful labelings to produce designs that possess special properties. In particular designs such as whist tournaments and generalized whist tournaments.

Another Anti-Oberwolfach Solution: Pancomponented 2-factorizations of complete graphs

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We pose and solve the existence of 2-factorizations of complete graphs and complete bipartite graphs that have the number of cycles per 2-factor varying, called *pancomponented*. Such 2-factorizations exist for all such graphs. The pancomponented problem requires a slight generalization of the methods used to solve pancyclic 2-factorization problem, by building 2-factors from cyclically generated 1-factors. These two solutions are offered as the basic approaches to constructing the two essential parameters of a 2-factorization: the size and the number of cycles in the 2-factors. They offer a strategic demonstration of the approaches to diverse problems.

Directed paths within n-gons

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Given n and t lengths $1 \leq l_1 < l_2 < \ldots < l_t \leq n-1$ of oriented diagonals within an n-gon. It is a problem of Brian Alspach to find a directed path within an n-gon using each length exactly once. Some partial results are presented (common work with Jens-P. Bode).

Metric and Partition Dimension

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We let $d(u,v)$ represent the distance between vertices u and v. Given a graph G, where S is a nonempty subset of $V(G)$, we say S is *resolving* if for each pair of distinct vertices u and v in G there is a vertex x in S where $d(u,x) \neq d(v,x)$. The minimum cardinality of all resolving sets is the *metric dimension* of G. Given w, a vertex of G, the distance from w to S, denoted $d(w,S)$, is the minimum distance between w and the vertices of S. Further, given $P = P_1, P_2, \ldots, P_k$ an ordered partition of $V(G)$ we say P is resolving if for each pair of distinct vertices u and v in G there is a part P_i where $d(u,P_i) \neq d(v,P_i)$. The *partition dimension* is the minimum order of all resolving partitions. The metric dimension and partition dimension of G will be denoted $\dim M(G)$ and $\dim P(G)$ respectively.

For given k and d, we construct a graph H with partition dimension k having the property that any graph with partition dimension k and diameter d is a subgraph of H. By examination of these graphs, we note properties that all graphs with a fixed partition dimension must share. For example, we show that if G has order n and partition dimension p then

$$\dim M(G) \leq \left(1 - \frac{1}{p-1}\right) n + \frac{2^{p-1}}{p-1}.$$

We show that if G has partition dimension p then the chromatic number of G is at most 2^{p-1}.

Also, we form a construction that shows for all integers a and b with $3 \leq a \leq b+1$ there exists a graph G with a partition dimension of a and metric dimension of b, answering a question of Chartrand, Salehi, and Zhang.

On a decomposition of complete graphs

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We show that for any prime power q, the complete graph on $1+q+q^2+q^3+q^4+q^5$ vertices can be decomposed into a union of $q(1+q)/2$ Siamese

$$SRG(1+q+q^2+q^3+q^4+q^5, q+q^2+q^3+q^4, -1+q+q^2+q^3, 1+q+q^2+q^3)$$

in such a way that every pair of the strongly regular graphs share $1+q^3$ disjoint cliques of size $1+q+q^2$ and a "little more" which may vary between the pairs. We use the lines of a projective plane of order q in our construction and explain the shape of the shared graph between each pair of the strongly regular graphs in terms of the incidence relation of the lines of the projective plane.

On two decomposition problems

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In the first part of the talk we deal with an extension of the Oberwolfach problem, the so-called Hamilton-Waterloo problem. This problem asks for a 2-factorization of the complete graph $K(2n+1)$ in which r of its 2-factors are isomorphic to a given 2-factor Q and s of its 2-factors are isomorphic to a given 2-factor R, with $r+s=n$. Some partial results on the problem will be presented.

During a stay of mine at S.F.U. Brian Alspach drew my attention to the following problem. Let $QD(n)$ be a digraph obtained from the n-dimensional cube $Q(n)$ by replacing each edge of $Q(n)$ with a pair of oppositely oriented arcs. Is there a decomposition of $QD(n)$ into Hamiltonian cycles? It is easy to see that such decomposition exists for all n even. The case of odd n will be discussed.

Colorings of Plane Graphs with no Rainbow Faces

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For a given coloring of a plane graph, a face is rainbow if all of its vertices are colored differently. A vertex coloring of a plane graph is a rainbow-free if no face is rainbow. R. Ramamurthi and D. West conjectured that if G is a triangle-free plane graph with $n \geq 4$ vertices, then

$$\chi_f(G) \geq \lceil n/2 \rceil + 1,$$

where $\chi_f(G)$ is the maximum number of colors used in a rainbow-free coloring of G. We answer the conjecture in affirmative. Moreover, we prove that for a plane graph with girth $g \geq 5$ there is a rainbow-free coloring with at least $\left\lceil \frac{g-3}{g-2}n - \frac{g-7}{2(g-2)} \right\rceil$ colors, if g is odd, and with at least $\left\lceil \frac{g-3}{g-2}n - \frac{g-6}{2(g-2)} \right\rceil$ colors, if g is even. The bounds are tight for all pairs n and g, $g \geq 4$ and $n \geq 5g/2-3$.

Hamiltonian decompositions of prisms over cubic graphs

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We discuss some results related to the following conjecture of Alspach and Rosenfeld: The prism over any 3-connected cubic graph has a decomposition into two Hamilton cycles. (The prism over a graph G consists of two disjoint copies of G and edges joining the two copies of each vertex of G.)

Edge-labellings of K_n with constant-weight on k-factors

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Is it possible to label the edges of K_n with distinct integer weights so that every Hamilton cycle has the same total weight? We give a local condition characterizing the labellings that witness this question's affirmative answer. Our characterization also addresses the question that arises when "Hamilton cycle" is replaced by "k-factor" for positive integers k. This is joint work with Scott Jones, Bojan Mohar, and Wal Wallis.

Square-free colorings of graphs

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Let G be a graph and c a coloring of its edges. If the sequence of colors along a walk of G is of the form $a_1,\ldots,a_n,a_1,\ldots,a_n$, the walk is called a square walk. We say that the coloring c is square-free if any open walk is not a square and call the minimum number of colors needed so that G has a square-free coloring a walk Thue number and denote it by $\pi_w(G)$. This concept is a variation of the Thue number introduced in: [1] N. Alon, J. Grytczuk, M. Hałuszczak, and O. Riordan, Non-repetitive colorings of graphs, Random Structures Algorithms 21 (2002) 336–346.

Using the walk Thue number several results of [1] are extended. The Thue number of some complete graphs is extended to Hamming graphs. This result (for the case of hypercubes) is used to show that if a graph G on n vertices and m edges is the subdivision graph of some graph, then $\pi_w(G) \leq n - \frac{m}{2}$. Graph products are also considered. An inequality for the Thue number of the Cartesian product of trees is extended to arbitrary graphs and upper bounds for the (walk) Thue number of the direct and the strong products are also given. Using the latter results the (walk) Thue number of complete multipartite graphs is bounded which in turn gives a bound for arbitrary graphs in general and for perfect graphs in particular.

Group chromatic number of planar graphs of girth at least 4

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Let A denote an Abelian group and $F(G,A)$ denote the set of all functions from $E(G)$ to A. For $f \in F(G,A)$, an (A, f)-coloring of G under the orientation D is a function $c : V(G) \mapsto A$ such that for every directed edge $e = uv, c(u) - c(v) \neq f(uv)$; G is A-*colorable* under the orientation D if for any function $f \in F(G,A)$, G has an (A, f)-coloring. It is known that whether G is A-colorable is independent of the choice of the orientation. The *group chromatic number* of a graph G is defined to be the smallest positive integer m for which G is A-colorable for any Abelian group A of order at least m under a given orientation D, and is denoted by $\chi_g(G)$. Jeager, Linial, Payan and Tarsi conjectured that every 5-edge connected graph is Z_3-connected.

In this talk we present a result of group coloring. As a special case, we show that Jeager's conjecture holds for planar graph.

Dudeney's Round Table Problem

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Dudeney's round table problem is a problem of constructing a uniform covering of the 2-paths by Hamilton cycles in the complete graph K_n. It was proposed about one hundred years ago and it is still unsettled. It is solved only when n is even, $n = p+2$ where p is an odd prime such that 2 or -2 is a primitive root of $GF(p)$, and some other cases. In this talk, we will survey the problem and show some new results.

On hamiltonicity of endo-Cayley digraphs

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Given a finite abelian group A, a subset $\Delta \subseteq A$ and an endomorphism ϕ of A, the endo-Cayley digraph $G_A(\phi, \Delta)$ is defined by taking A as vertex set and every vertex x adjacent to the vertices $\phi(x) + a$ with $a \in \Delta$. When A is cyclic and the set Δ is of the form $\Delta = \{e, e+h, \ldots, e+(d-1)h\}$, the digraph G is called a consecutive digraph. In this paper we study the hamiltonicity of endo-Cayley digraphs by using three techniques: line digraph, merging cycles and a generalization of the factor group lemma. The results are applied to consecutive digraphs.

On the oriented flow number of rank 3 orientable matroids

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The oriented flow number of an orientable matroid, $\phi_o(M)$, is the extension of the (dual) notion of the circular chromatic number of a graph. Goddyn, Hliněný, and Hochstättler have shown $\phi_o(M) \leq 14r^2 \ln r$, where r is the rank of the oriented matroid M. In addition, they've show $\phi_o(M) \leq 17$ for rank 3 oriented matroids. We show first show $\phi_o(M) \leq 4$ for rank 3 uniform matroids M and then extend this result to the general rank 3 case.

Construction of Graphs with Maximum Graphical Structure and Tenacity T

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The tenacity of a graph G, $T(G)$, is defined by $T(G) = \min\{\frac{|A|+\tau(G-A)}{\omega(G-A)}\}$, where the minimum is taken over all vertex cutset A of G. We define $G-A$ to be the graph induced by the vertices of $V-A$, $\tau(G-A)$ is the number of vertices in the largest component of the graph by $G-A$ and $\omega(G-A)$ is the number of components of $G-A$.

In this paper, the maximum graphical structure is obtained when the number of vertices p of a connected graph G and tenacity $T(G) = T$ are given. Finally the method of constructing the sort of graphs are also presented.

Uniform coverings of 2-paths with 5-paths in the complete graph

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A uniform covering of the 2-paths in K_n with k-paths (k-cycles) is a set S of k-paths (k-cycles) having the property that each 2-path in K_n lies in exactly one k-path (k-cycle) in S. Only the following cases of the problem of constructing a uniform covering of the 2-paths in K_n with k-paths or k-cycles have been solved: with 3-cycles, 3-paths, 4-cycles and 4-paths.

In this talk, we solve the problem in the case of 5-paths, that is, we prove that there exists a uniform covering of 2-paths with 5-paths in K_n if and only if n is even or $n \equiv 1 \pmod 8$, where $n \geq 6$.

Minimum Chromaticity and Efficient Domination of Circulant Graphs

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We present results for two related problems on circulant graphs. We determine families of circulant graphs for which each graph $G = G^c(n;S)$ has $\chi(G) \leq 3$, and we determine when G admits efficient domination. In particular, for the chromaticity problem, we show that when $S = \left\{s_1, s_2, \ldots, s_k | s_k > s_{k-1} > \cdots > s_1 \wedge s_1 > \left\lceil \frac{s_k}{2} \right\rceil \right\}$ or $S = \{s_1, s_2 | s_2 \neq 2s_1\}$, there exists an n_0 such that $\forall n \geq n_0$, $\chi(G) \leq 3$. We also prove that $\chi(G) \leq 3$ for every recursive circulant graph $G = RG^c(n;d)$, $n = c \cdot d^m$, $1 \leq c \leq d$. For the efficient domination problem, we give a new approach for determining necessary and sufficient conditions for a circulant graph G to admit efficient domination. We derive a closed form solution for the number of different sets S which, for a given n, yield an efficient dominating set of $G = G^c(n;S)$.

Pancyclic BIBD Block-Intersection Graphs

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Given a combinatorial design $\mathcal{D}$ with block set $\mathcal{B}$, its block-intersection graph $G_{\mathcal{D}}$ is the graph having vertex set $\mathcal{B}$ such that two vertices b_1 and b_2 are adjacent if and only if b_1 and b_2 have non-empty intersection. Alspach and Hare proved that if $\mathcal{D}$ is a balanced incomplete block design, BIBD$(v,k,1)$ with $k \geq 3$, then $G_{\mathcal{D}}$ is edge-pancyclic (i.e. each edge of $G_{\mathcal{D}}$ is contained in a cycle of each length $\ell = 3,4,\ldots,|V(G_{\mathcal{D}})|$). In this paper, it is proved that if $\mathcal{D}$ is a BIBD(v,k,λ) with arbitrary index $\lambda \geq 2$, then $G_{\mathcal{D}}$ is pancyclic (i.e. $G_{\mathcal{D}}$ contains a cycle of each length $\ell = 3,4,\ldots,|V(G_{\mathcal{D}})|$).

This is joint work with Aygul Mamut (Xinjiang University, China) and Michael Raines (Western Michigan University).

Some Hamilton Path Problems

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An edge in a graph is called traceable if it is contained in a Hamilton path or non-traceable if it not contained in any Hamilton path. Extremum results and problems concerning the decomposition of graphs into traceable and non-traceable edges will be presented. The existence of Hamilton paths in a special class of graphs will also be mentioned.

Maximal sets of triangle-factors

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A triangle-factor in a graph G is a vertex-disjoint union of triangles (K_3s) which spans G. A collection of edge-disjoint triangle-factors in K_n is *maximal* if the complement of the graph G formed by the union of these triangle-factors does not contain a triangle-factor. The problem of determining the spectrum $\{k$: there exists a maximal set of k triangle-factors on n vertices $\}$ has been essentially solved for each $n \equiv 0 \pmod 3$, with the notable exception where k is the smallest integer not less than $n/6$ (the smallest possible value for k). We briefly examine this case.

Hamilton Cycles in Small Cubic Cayley Graphs

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We report on some exhaustive computer searches for Hamilton cycles in small cubic Cayley graphs and in small two-in two-out Cayley digraphs. One perplexing Cayley graph of 10080 vertices has resisted our attempts to show that it is Hamiltonian. (Scott Effler, Brendan McKay, Alex Hertel, Philip Hertel, and Ian Shields all contributed to the searches.)

IC-Coloring of Graphs

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Given a coloring $f : V(G) \to \mathbb{N}$ of graph G and any subgraph $H \subset G$ we define $f_s(H) = \sum_{v \in V(H)} f(v)$. In particular, we denote $f_s(G)$ by $S(f)$. The coloring f is called an IC-coloring if for any integer $k \in [1, S(f)]$ there is a connected subgraph $H \subset G$ such that $f_s(H) = k$. Also, we define the maximum index of G to be

$$M(G) = \max\{\ S(f)\ :\ f \text{ is an IC-coloring of } G\ \}.$$

In this talk we will examine some well-known classes of graphs and will determine their maximum indices.

Cyclic, Diagonal, and Facial Colorings

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A cyclic coloring of a graph on a surface is a coloring of its vertices such that every two vertices incident with the same face receive distinct colors. The diagonal coloring is a generalization of the cyclic coloring. In the talk, we introduce the facial coloring, which is also generalization of the cyclic coloring. We present some new results and problems about these kinds of colorings.

Permanents and Graphs

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In this paper the sufficient condition for a graph to be Hamiltonian and non-Hamiltonian are discussed in terms of permanent. Some simple results on tree with permanent of $A(T)$ and $X^1(T)$, like permanental values of $A(T)$ and $X^1(G)$, upper bound of number of 1-factor subgraphs and necessay condition of tree separable graphs. If G is a bipartite graph with $\#X = \#Y = n$ such that per $A(G) = (n!)^2$, then G is a complete bipartite graph. Also some results in permanent of $A(G)$ of separable graphs, equivalence and partial order relations are discussed.

Semidirect Product of Association Schemes

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Let (X,H) and (Y,K) be association schemes, and let π be a group homomorphism from $H//O^{\theta}(H)$ to Aut(K). Relative to π, an external semidirect product of (X,H) and (Y,K) is defined. In this talk, we generalize this construction by defining a fusion scheme of the semidirect product for each π-invariant normal closed subset N of K. Our purpose is to show that each direct product, each wreath product, and each semidirect product arises from our construction when π and N are chosen appropriately. We also show that our construction produces association schemes which are neither direct products, nor wreath products, nor semidirect products of two given association schemes. This is based on joint work with S. Bang and M. Hirasaka.

Edge-disjoint cycles through prescribed vertices

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We present a sufficient condition for the existence of edge-disjoint cycles, each containing a prescribed set of vertices in a graph. This generalizes several previously known Ore-type results. More precisely, we have the following theorem.

Let W be a subset of the vertices of a graph G of order n, and let $k \geq 0$ be an integer. Suppose the induced graph $G[W]$ is $2(k+1)$-connected, and that for any two vertices having distance two in $G[W]$, at least one of the two has degree at least $n/2+2k$ in G. Then G contains $k+1$ pairwise edge-disjoint cycles where each cycle contains all vertices in W.

A main lemma is a Hamilton-cycle packing result regarding complements of bipartite graphs: Let G be a $2k$-connected graph and let $G[X]$ and $G[Y]$ be cliques for some partition $V(G) = X \cup Y$. Then G has k Hamilton cycles $C_1, \ldots, C_k$ such that

$$E(C_i) \cap E(C_j) \subseteq E(G[Y]) \text{ for } i \neq j.$$

This lemma may be of great interest on its own; if $G = K_{2k+1}$, and Y is a single vertex, then it becomes the well-know result on decomposition of K_{2k+1} into k edge-disjoint Hamilton cycles.

List M-partition Problems for Directed Graphs

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The list M-partition problem is the general form of the list coloring and list homomorphism problems. Each matrix over 0,1,* defines a list M-partition problem for directed graphs. Let M be a 3-by-3 matrix over 0,1,*. Given a digraph, G, and a set $L(v) \subset \{a_1, a_2, a_3\}$ for each vertex v in G, the list M-partition problem asks if G can be partitioned into 3 sets, a_1, a_2, a_3, where each vertex, v, can only be placed in a part that is in $L(v)$ and the following conditions hold for each set a_i:

1. Part a_i is stable when $M_{ii} = 0$
2. Part a_i is a clique when $M_{ii} = 1$
3. If $M_{ij} = 0$ then there are no arcs from part a_i to part a_j
4. If $M_{ij} = 1$ then there are all possible arcs from part a_i to part a_j

In this talk I will show that if M is a 3-by-3 matrix over 0,* then the list M-partition problem has a polynomial solution in all but three cases. Two of the three cases are well known NP-complete problems, 3-coloring and finding a stable cut-set. I will show that third case is NP-complete as well, it will be referred to as the 3-cycle partition. A set of tools was developed by Feder, Hell, Klein, and Motwani to completely classify the complexity of list M-partition problems when M is a small, symmetric matrix (a symmetric matrix corresponds to the undirected version of the list M-partition). A similar classification of complexity for list M-partition problems where M is any 3x3 matrix over 0,1,* has almost been completed. In this analysis I rely on some of the same tools that were used in the undirected analysis, however some problems prove to be more challenging and new techniques must be used. This increased complexity is evident in the material presented during this talk. In the undirected case the only NP-complete list M-partitions when M is a 3x3 matrix are 3-colouring and finding a stable cut-set. In the directed case (when M is 3x3), there is a third NP-complete problem, the 3-cycle partition discussed above.

Fractional biclique covers and partitions of graphs

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A biclique is a complete bipartite subgraph of a graph G. The biclique cover number, $bc(G)$ and the biclique partition number, $bp(G)$, are the minimum number of bicliques needed to cover and partition, respectively, the edges of G. Since $bc(G)$ and $bp(G)$ are integer-valued graph parameters, they may be viewed as integer programs. This talk will look at the linear relaxation of these integer programs and present linear programs which define the fractional biclique cover number, $bc^*(G)$, and the fractional biclique partition number, $bp^*(G)$. Also, this talk will observe that $bc^*(G)$ and $bp^*(G)$ provide lower bounds on $bc(G)$ and $bp(G)$ respectively, and conditions for equality will be given. Further, corresponding fractional analogues for many results about $bc(G)$ and $bp(G)$ will be presented.

On the degree monotonicity of cages

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A $(k;g)$-graph is a k-regular graph with girth g. A $(k;g)$-cage is a $(k;g)$-graph with the *least* number of vertices. The order of a $(k;g)$-cage is denoted by $f(k;g)$. In this paper we show that $f(k+2;g) \geq f(k;g)$ for $k \geq 2$ and present some partial results to support the conjecture that $f(k_1;g) < f(k_2;g)$ if $k_1 < k_2$.