Computer Science 1820

Solutions for Recommended Exercises

Section 1.3

---

6. (a) "$\exists$" means "there exists," so $\exists x N(x)$ means that there exists some member x of the population for which $N(x)$ is true. In English, this quantification is "Some student in my school has visited North Dakota."

   (b) "$\forall$" means "for all," so $\forall x N(x)$ means that $N(x)$ is true for every member x of the population. In English, this quantification is "Every student in my school has visited North Dakota."

   (c) "It is not the case that some student in my school has visited North Dakota."

   (d) "Some student in my school has not visited North Dakota."

   (e) "It is not the case that every student in my school has visited North Dakota."

   (f) "Every student in my school has not visited North Dakota."

8. (a) "Every rabbit hops."

   (b) "Every animal is a rabbit and hops," or "Every animal is a hopping rabbit."

   (c) "Some rabbit hops."

   (d) "Some animal is a rabbit and hops," or "Some animal is a hopping rabbit."

10. (a) $\exists x(C(x) \land D(x) \land F(x))$ (b) $\forall x(C(x) \lor D(x) \lor F(x))$

    (c) $\exists x(C(x) \land F(x) \land (\neg D(x)))$ (d) $\neg\exists x(C(x) \land D(x) \land F(x))$

    (e) $(\exists x C(x)) \land (\exists x D(x)) \land (\exists x C(x))$, or $(\exists x C(x)) \land (\exists y D(y)) \land (\exists z C(z))$

12. (a) $(0)+1=1$, $2(0)=0$, and $1>0$, so $Q(0)$ is $\boxed{\textit{true.}}$

(b) $(-1)+1=0$, $2(-1)=-2$, and $0>-2$, so $Q(-1)$ is $\boxed{\textit{true.}}$

(c) $(1)+1=2$, $2(1)=2$, but $2 \not> 2$, so $Q(1)$ is $\boxed{\textit{false.}}$

(d) There *does* exist an integer x for which $Q(x)$ is true: 0! So, $\exists x Q(x)$ is $\boxed{\textit{true.}}$

(e) $Q(x)$ is *not* true for all integers x; specifically, $Q(1)$ is false! So, $\forall x Q(x)$ is $\boxed{\textit{false.}}$

(f) There does exist an integer x for which $Q(x)$ is false (1 is such an integer), so there exists an integer for which $\neg Q(x)$ is true. Ergo, $\exists x \neg Q(x)$ is $\boxed{\textit{true.}}$

(g) $Q(x)$ is not false for every integer x ($Q(0)$ is not false), so $\neg Q(x)$ is not true for every integer x. Hence, $\forall x \neg Q(x)$ is $\boxed{\textit{false.}}$

14. (a) $(-1)^3=-1$, so there exists a real number x for which $x^3=-1$ is true; thus, the given statement is $\boxed{\textit{true.}}$

(b) $(0.5)^4=0.0625$, $(0.5)^2=0.25$, and $0.0625<0.25$, so $x=0.5$ makes $x^4<x^2$ true; therefore, the given statement is $\boxed{\textit{true.}}$

(c) Let x be any real number. Then, $(-x)^2=(-1)^2x^2=(1)x^2=x^2$, so $(-x)^2=x^2$ is true for every real number. Consequently, the given statement is $\boxed{\textit{true.}}$

(d) $2(-1)=-2$, but $-2 \not> -1$, so $2x>x$ is *not* true for every real number x; ergo, the given statement is $\boxed{\textit{false.}}$

18. (a) $P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2)$

(b) $P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2)$

(c) $(\neg P(-2)) \vee (\neg P(-1)) \vee (\neg P(0)) \vee (\neg P(1)) \vee (\neg P(2))$

(d) $(\neg P(-2)) \wedge (\neg P(-1)) \wedge (\neg P(0)) \wedge (\neg P(1)) \wedge (\neg P(2))$

(continued)

(continued)

(e) $\neg(P(-2) \vee P(-1) \vee P(0) \vee P(1) \vee P(2))$

Using De Morgan's Law, this becomes: $(\neg P(-2)) \wedge (\neg P(-1)) \wedge (\neg P(0)) \wedge (\neg P(1)) \wedge (\neg P(2))$

(f) $\neq (P(-2) \wedge P(-1) \wedge P(0) \wedge P(1) \wedge P(2))$

Using De Morgan's Law, this becomes: $(\neg P(-2)) \vee (\neg P(-1)) \vee (\neg P(0)) \vee (\neg P(1)) \vee (\neg P(2))$

20. (a) $P(-5) \vee P(-3) \vee P(-1) \vee P(1) \vee P(3) \vee P(5)$

(b) $P(-5) \wedge P(-3) \wedge P(-1) \wedge P(1) \wedge P(3) \wedge P(5)$

(c) We can express the given quantified statement in English as "For every x that is not 1, $P(x)$ is true," which in turn allows us to express it as

$$P(-5) \wedge P(-3) \wedge P(-1) \wedge P(3) \wedge P(5).$$

(d) We can express the given quantified statement in English as "There is some x that is nonnegative for which $P(x)$ is true," which is logically equivalent to $P(1) \vee P(3) \vee P(5)$.

(e) We express the given statement in English as "There is some x for which $P(x)$ is not true, and for every negative x, $P(x)$ is true." Note the consequence of this conjunction: whatever value of x makes $P(x)$ false must be nonnegative! Ergo, the statement becomes

$$P(-5) \wedge P(-3) \wedge P(-1) \wedge [(\neg P(1)) \vee (\neg P(3)) \vee (\neg P(5))].$$

30. (a) $P(1,3) \vee P(2,3) \vee P(3,3)$

(b) $P(1,1) \wedge P(1,2) \wedge P(1,3)$

(c) $(\neg P(2,1)) \vee (\neg P(2,2)) \vee (\neg P(2,3))$

(d) $(\neg P(1,2)) \wedge (\neg P(2,2)) \wedge (\neg P(3,2))$

32. To begin, let the domain in each part of this exercise be the set of all (living) animals. Also, we will use these four logical equivalences:

$$\neg(\forall x P(x)) \equiv \exists x \neg P(x),\ \ \neg(\exists x P(x)) \equiv \forall x \neg P(x),\ \ \neg(A \to B) \equiv (A \wedge \neg B),\ \text{ and }\ \neg(A \wedge B) \equiv (A \to \neg B).$$

   (a) We let $D(x)$ represent "x is a dog" and let $F(x)$ denote "x has fleas." Then, the original statement is $\forall x(D(x) \to F(x))$. Its negation is therefore

$$\neg[\forall x(D(x) \to F(x))] \equiv \exists x \neg(D(x) \to F(x)) \equiv \boxed{\exists x(D(x) \wedge \neg F(x)).}$$

   In English, this negation is "Some dog does not have fleas."

   (b) Letting $H(x)$ mean "x is a horse" and $A(x)$ stand for "x can add," the original statement becomes $\exists x(H(x) \wedge A(x))$. Its negation is

$$\neg[\exists x(H(x) \wedge A(x))] \equiv \forall x \neg(H(x) \wedge A(x)) \equiv \boxed{\forall x(H(x) \to \neg A(x)),}$$

   whose English form is "Every horse cannot add."

   (c) Letting $K(x)$ be "x is a koala" and $C(x)$ be "x can climb," the original statement is $\forall x(K(x) \to C(x))$. Its negation is

$$\neg[\forall x(K(x) \to C(x))] \equiv \exists x \neg(K(x) \to C(x)) \equiv \boxed{\exists x(K(x) \wedge \neg C(x)),}$$

   whose English form is "Some koala cannot climb."

   (d) Letting $M(x)$ represent "x is a monkey" and $F(x)$ denote "x can speak French," the original statement is $\neg \exists x(M(x) \wedge F(x))$. Its negation is

$$\neg[\neg \exists x(M(x) \wedge F(x))] \equiv \boxed{\exists x(M(x) \wedge F(x)),}$$

   whose English form is "Some monkey can speak French."

(continued)

(continued)

(e) Letting $P(x)$ be "x is a pig," $S(x)$ be "x can swim," and $C(x)$ be "x can catch fish," the original statement is $\exists x(P(x) \wedge (S(x) \wedge C(x)))$. Its negation is

$$\neg[\exists x(P(x) \wedge (S(x) \wedge C(x)))] \equiv \forall x \neg(P(x) \wedge (S(x) \wedge C(x)))$$

$$\equiv \forall x(P(x) \rightarrow \neg(S(x) \wedge C(x))) \equiv \boxed{\forall x(P(x) \rightarrow (\neg S(x) \vee \neg C(x))),}$$

whose English form is "Every pig cannot cannot swim or cannot catch fish."

34. (a) Let the domain be the set of all drivers, and let $O(x)$ represent "x obeys the speed limit." Then, the original statement is $\exists x \neg O(x)$, and its negation is

$$\neg[\exists x \neg O(x)] \equiv \forall x \neg(\neg O(x)) \equiv \boxed{\forall x O(x).}$$

In English, this negation is "Every driver obeys the speed limit."

(b) Let the domain be the set of all movies, $W(x)$ denote "x is Swedish," and $S(x)$ mean "x is serious." Then, the original statement is $\forall x(W(x) \rightarrow S(x))$, so its negation is

$$\neg[\forall x(W(x) \rightarrow S(x))] \equiv \exists x \neg(\forall x(W(x) \rightarrow S(x))) \equiv \boxed{\exists x(W(x) \wedge \neg S(x)).}$$

In English, this negation is "Some Swedish movie is not serious."

(c) Letting the domain be the set of all (living) persons and $K(x)$ represent "x can keep a secret," the original statement becomes $\neg \exists x K(x)$. Then, its negation is

$$\neg[\neg \exists x K(x)] \equiv \boxed{\exists x K(x),}$$

whose English form is "Someone can keep a secret."

(continued)

(continued)

(d) Let the domain be the set of people in this class, and let $G(x)$ be "x has a good attitude." Then, the original statement is $\exists x \neg G(x)$, so its negation is

$$\neg[\exists x \neg G(x)] \equiv \forall x \neg(\neg G(x)) \equiv \boxed{\forall x G(x).}$$

In English, this negation is "Every student in this class has a good attitude."

36. (a) 0 or 1 (b) $-\sqrt{2}$ or $\sqrt{2}$ (c) 0