Computer Science 1820

Solutions for Recommended Exercises

Section 1.4

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2. (a) "There is some number x whose product with every number y is y."

   (b) "If x is any nonnegative number and y is any negative number, then $x - y$ is positive."

   (c) "For any pair of numbers x and y there exists a number z such that $x = y + z$."

4. (a) "Some student has taken some class."

   (b) "Some student has taken every class."

   (c) "Every student has taken some class."

   (d) "Some class has been taken by every student."

   (e) "Every class has been taken by some student."

   (f) "Every student has taken every class.

10. (a) $\forall x F(x, \text{Fred})$

    (b) $\forall y F(\text{Evelyn}, y)$

    (c) $\forall x \exists y F(x, y)$

    (d) $\neg \exists x \forall y F(x, y)$

    (e) $\forall y \exists x F(x, y)$

    (f) $\neg \exists x (F(x, \text{Fred}) \wedge F(x, \text{Jerry}))$

(continued)

(continued)

(g) $\exists x \exists y(x \neq y \wedge F(\text{Nancy}, x) \wedge F(\text{Nancy}, y) \wedge \forall z[F(\text{Nancy}, z) \rightarrow (z = x \vee z = y)])$

(h) $\exists y(\forall x F(x,y) \wedge \forall b((\forall a F(a,b)) \rightarrow b = y))$

(i) $\neg \exists x F(x,x)$

(j) $\exists x \exists y(x \neq y \wedge F(x,y) \wedge \forall z(F(x,z) \rightarrow z = y))$

12. (a) $\neg I(\text{Jerry})$

(b) $\neg C(\text{Rachel}, \text{Chelsea})$

(c) $(\neg C(\text{Jan}, \text{Sharon})) \wedge (\neg C(\text{Sharon}, \text{Jan}))$

(d) $\neg \exists x C(x, \text{Bob})$

(e) $(\neg C(\text{Sanjay}, \text{Joseph})) \wedge \forall y(y \neq \text{Joseph} \rightarrow C(\text{Sanjay}, y))$

(f) $\exists x \neg I(x)$

(g) $\neg \forall x I(x)$

(h) $\exists x(I(x) \wedge \forall y(I(y) \rightarrow y = x))$

(i) $\exists x((\neg I(x)) \wedge \forall y(y \neq x \rightarrow I(y)))$

(j) $\forall x(I(x) \rightarrow (\exists y C(x,y)))$

(k) $\exists x(I(x) \wedge (\neg \exists y C(x,y)))$

(l) $\exists x \exists y((\neg C(x,y)) \wedge (\neg C(y,x)))$

(m) $\exists x \forall y C(x,y)$

(n) $\exists x \exists y(x \neq y \wedge \exists z \neg (C(x,z) \wedge C(y,z)))$

(o) $\exists x \exists y(x \neq y \wedge \forall z(C(x,z) \vee C(y,z)))$

28. (a) Every real number has square, so the given statement is $\boxed{\textit{true.}}$

(b) Not every real number has a square root (no real y satisfies $-1 = y^2$, for example), so the given statement is $\boxed{\textit{false.}}$

(c) There *is* a real number whose product with any real number is zero, and that's zero itself! So, the given statement is $\boxed{\textit{true.}}$

(d) The addition of real numbers is *commutative,* so the given statement is $\boxed{\textit{false.}}$

(e) Every nonzero real number has a *multiplicative inverse,* so the given statement is $\boxed{\textit{true.}}$

(f) There is *no* real number whose product with every nonzero real number is one, so the given statement is $\boxed{\textit{false.}}$

(g) $1 - x$ exists for every real number x, so the given statement is $\boxed{\textit{true.}}$

(h) The given statement is true if this system of equations has a solution:

$$\begin{array}{rcrcl} x & + & 2y & = & 2 \\ 2x & + & 4y & = & 5 \end{array}$$

Multiplying both sides of the first equation by 2 (which we can prove does not affect the solution) yields $2x + 4y = 4$, which is *inconsistent* with the second equation (no values of x and y can make $2x + 4y$ equal to 4 *and* 5 simultaneously). Ergo, the given statement is $\boxed{\textit{false.}}$

(i) The system of equations inside the given statement has a *unique* solution, $x = y = 1$, so that system is *not* satisfied by every real value of x. Hence, the given statement is $\boxed{\textit{false.}}$

(j) Every pair of real numbers (not necessarily distinct) has an *average value,* so the given statement is $\boxed{\textit{true.}}$

32. (a) $\neg\,[\exists z \forall y \forall x T(x,y,z)] \equiv \forall z \neg\,[\forall y \forall x T(x,y,z)] \equiv \forall z \exists y \neg\,[\forall x T(x,y,z)] \equiv \boxed{\forall z \exists y \exists x \neg T(x,y,z).}$

(continued)

(continued)

(b) $\neg[\exists x \exists y P(x,y) \land \forall x \forall y Q(x,y)] \equiv \neg[\exists x \exists y P(x,y)] \lor \neg[\forall x \forall y Q(x,y)]$

$$\equiv \forall x \neg[\exists y P(x,y)] \lor \exists x \neg[\forall y Q(x,y)] \equiv \boxed{\forall x \forall y \neg P(x,y) \lor \exists x \exists y \neg Q(x,y).}$$

(c) $\neg[\exists x \exists y (Q(x,y) \leftrightarrow Q(y,x))] \equiv \forall x \neg[\exists y (Q(x,y) \leftrightarrow Q(y,x))]$

$$\equiv \forall x \forall y \neg(Q(x,y) \leftrightarrow Q(y,x)) \equiv \boxed{\forall x \forall y (Q(x,y) \oplus Q(y,x)).}$$

(d) $\neg[\forall y \exists x \exists z (T(x,y,z) \lor Q(x,y))] \equiv \exists y \neg[\exists x \exists z (T(x,y,z) \lor Q(x,y))]$

$$\equiv \exists y \forall x \neg[\exists z (T(x,y,z) \lor Q(x,y))] \equiv \exists y \forall x \forall z \neg[T(x,y,z) \lor Q(x,y)]$$

$$\equiv \boxed{\exists y \forall x \forall z [(\neg T(x,y,z)) \land (\neg Q(x,y))].}$$

38. (a) Let the domain be the set of all students in this class, and let $L(x)$ be "x likes mathematics." Then, the given statement is $\forall x L(x)$ and its negation is $\exists x \neg L(x)$, whose English form is "Some student in this class does not like mathematics," *which is crazy talk!* (◡̈)

(b) Let the domain be the set of all students in this class, and let $S(x)$ be "x has seen a computer." Then, the given statement is $\exists x \neg S(x)$ and its negation is $\forall x S(x)$, whose English form is "Every student in this class has seen a computer."

(c) Let $T(x,y)$ represent "x has taken course y offered at this school" where the domain for x is the set of all students at this school and the domain for y is the set of all math courses offered by this school. Then, the given statement is $\exists x \forall y T(x,y)$ and its negation is $\forall x \exists y \neg T(x,y)$, whose English form is "Every student has not taken some mathematics course offered at this school."

(d) Let $I(x,y,z)$ denote "x has been in room y of building z on campus," where the domain for x is the set of all students attending this school, the domain for z is the set of all buildings on its campus, and the domain for y is the set of rooms in those buildings. Then, the given statement is $\exists x \forall z \exists y I(x,y,z)$ and its negation is $\forall x \exists z \forall y \neg I(x,y,z)$, or "Every student has not been in any room of some building on campus."